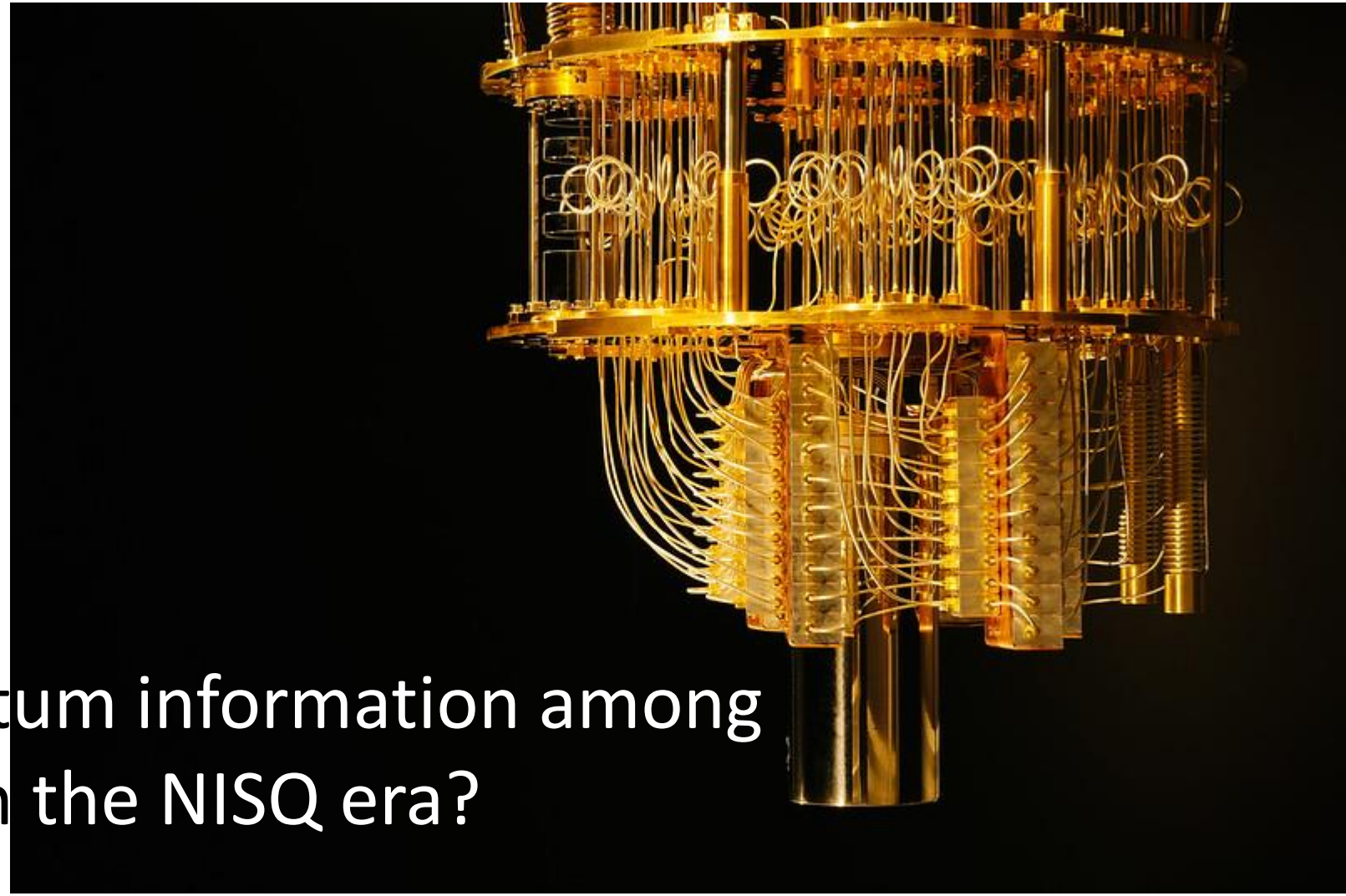
Multipartite entanglement distribution in quantum networks

E. Sutcliffe and A. Beghelli
Optical Networks Group

Thanks to UKRI Project “Quantum Data Centres of the Future” & Delivering Quantum Technologies UCL CDT

Images from flaticon.com

The fundamental problem



How to distribute quantum information among several users in the NISQ era?

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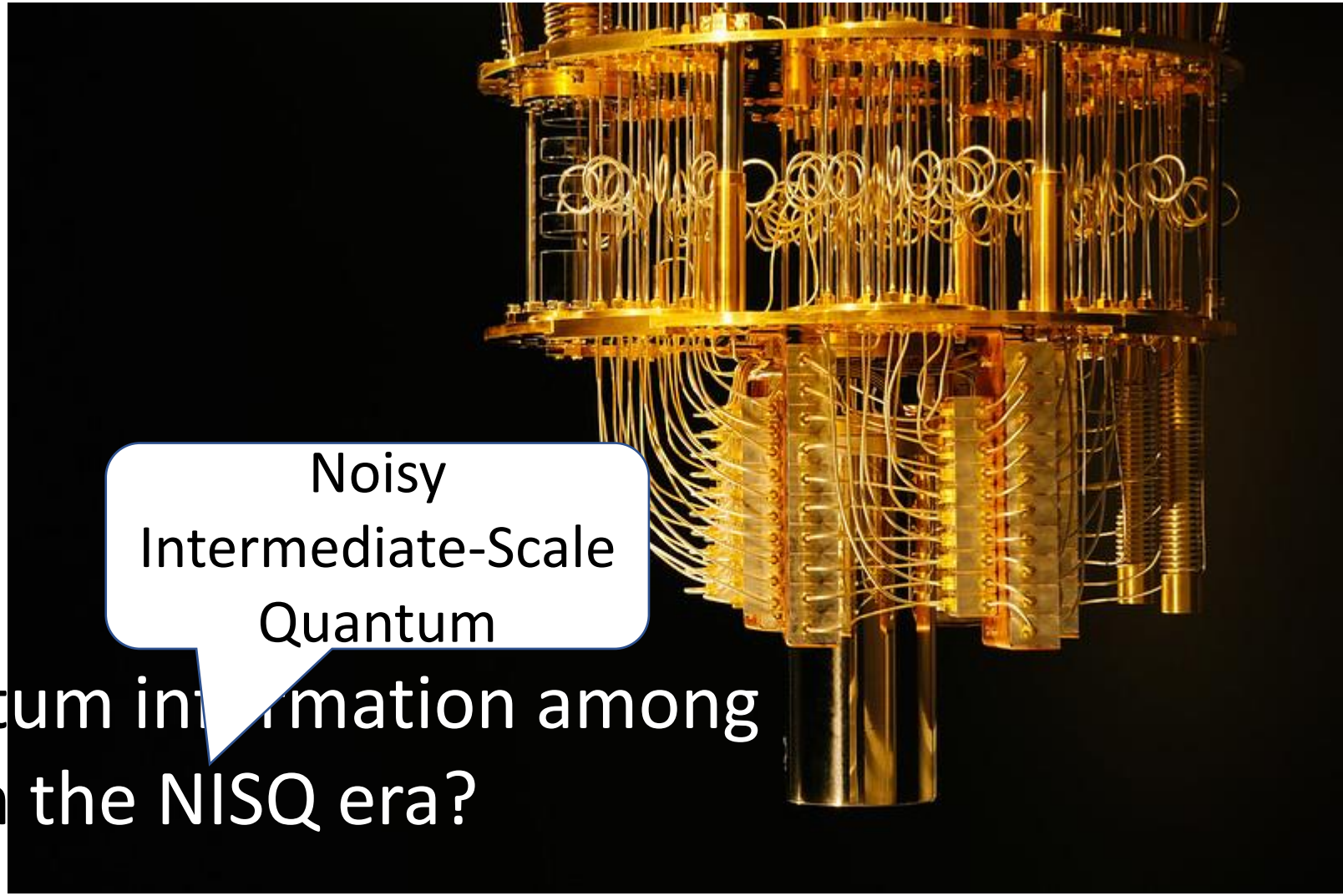
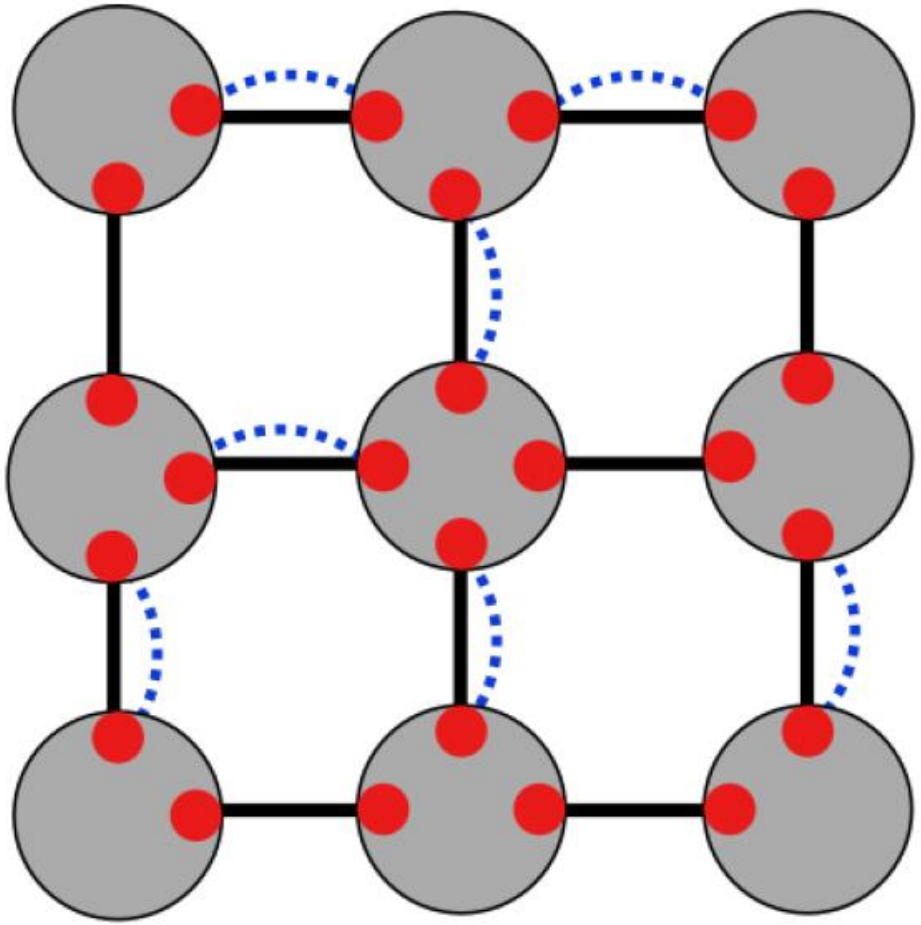


How to distribute quantum information among several users in the NISQ era?

The (potential) applications

- Distributed quantum computing
- Distributed quantum sensing
- Secret sharing

The network model



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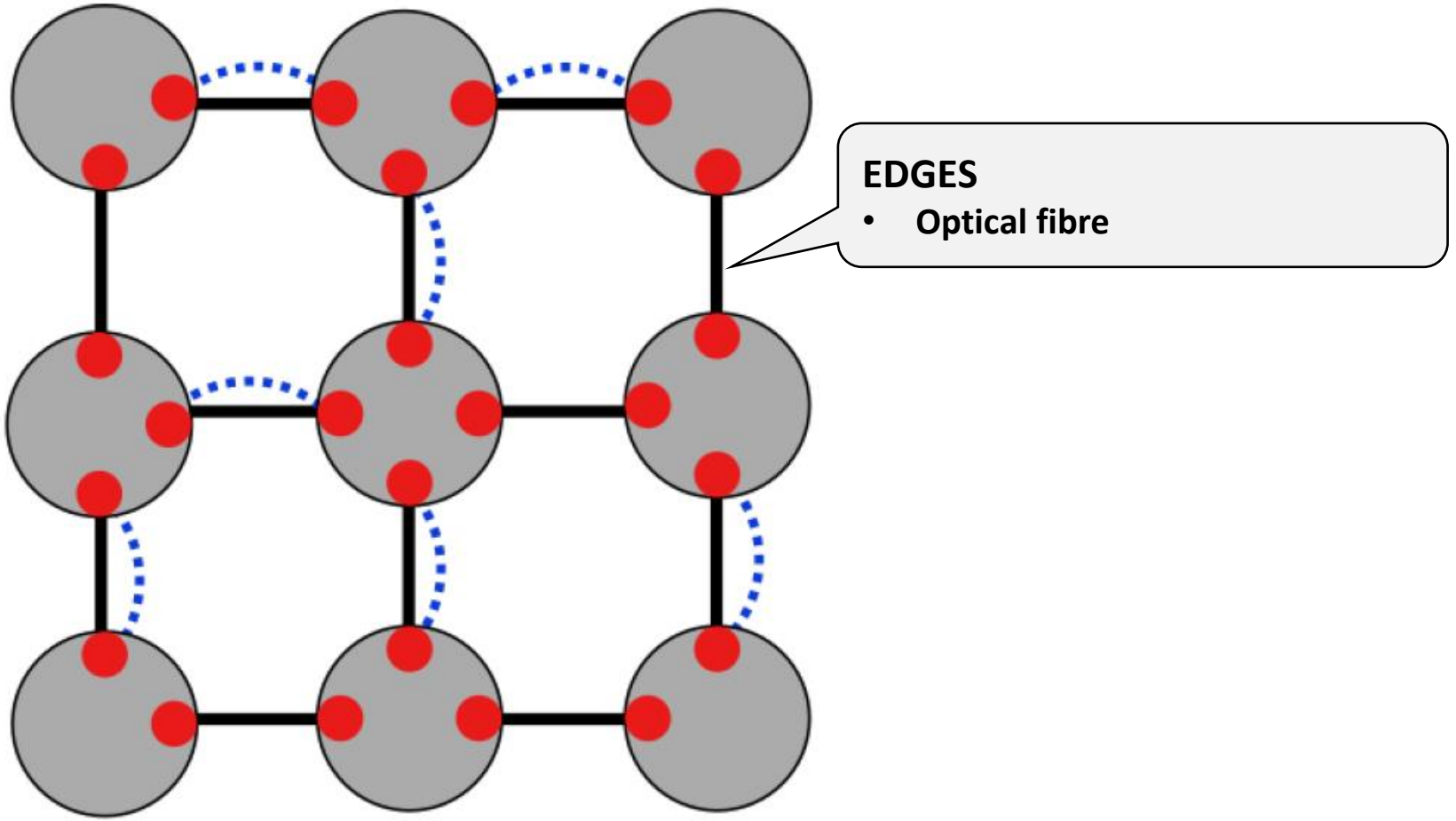
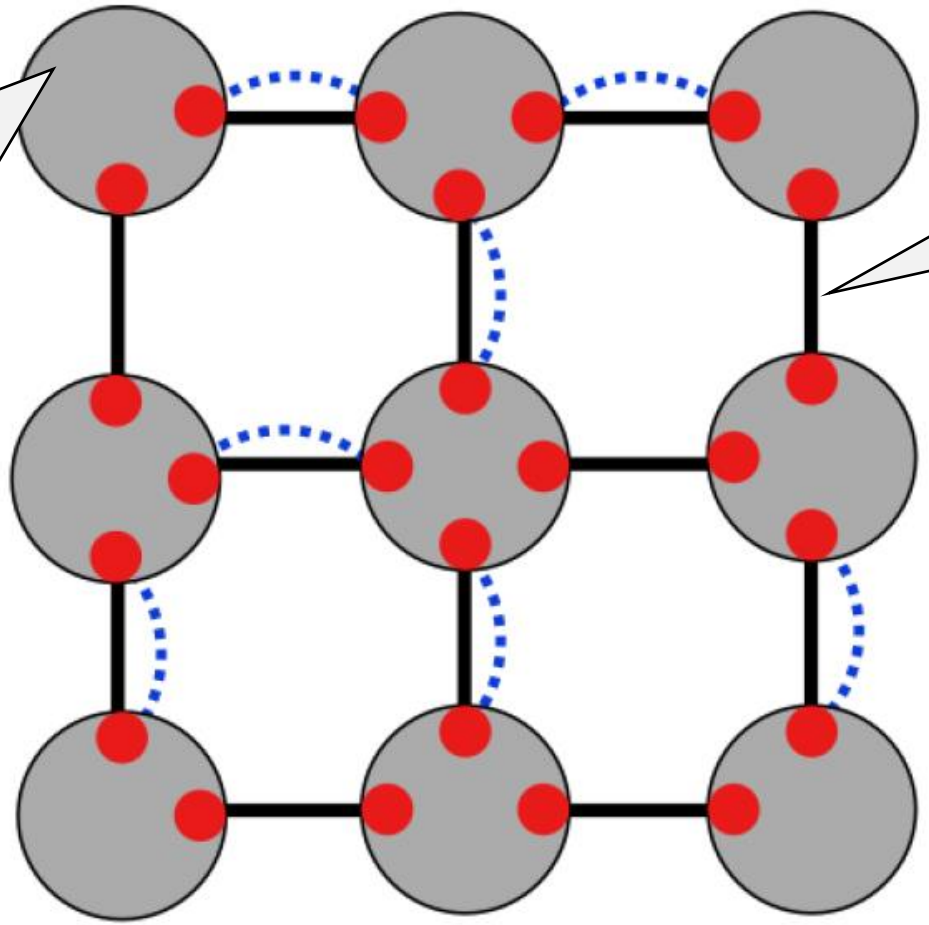


The network model

NODES
- **1 quantum memory per edge (T_c)**
- **Can act as quantum repeaters**
- **Can perform entanglement fusion**



EDGES
- **Optical fibre**

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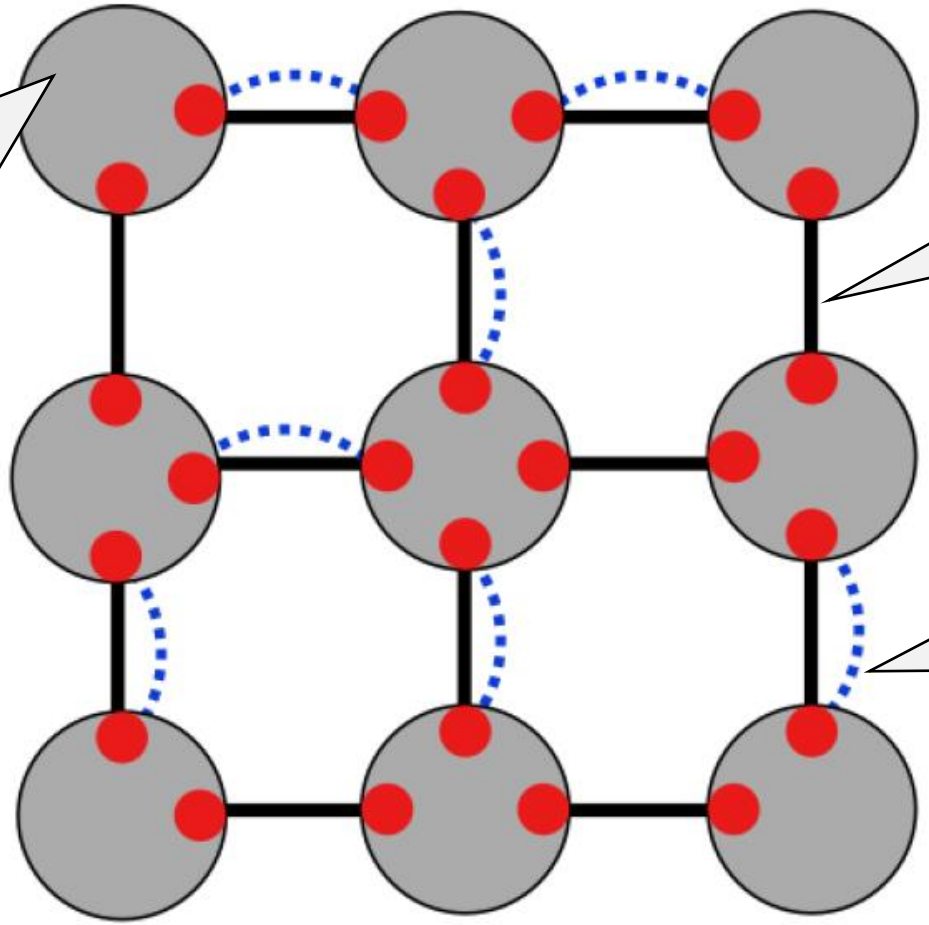

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Entanglement link
$p=p_{op}*(1-p_{loss})$

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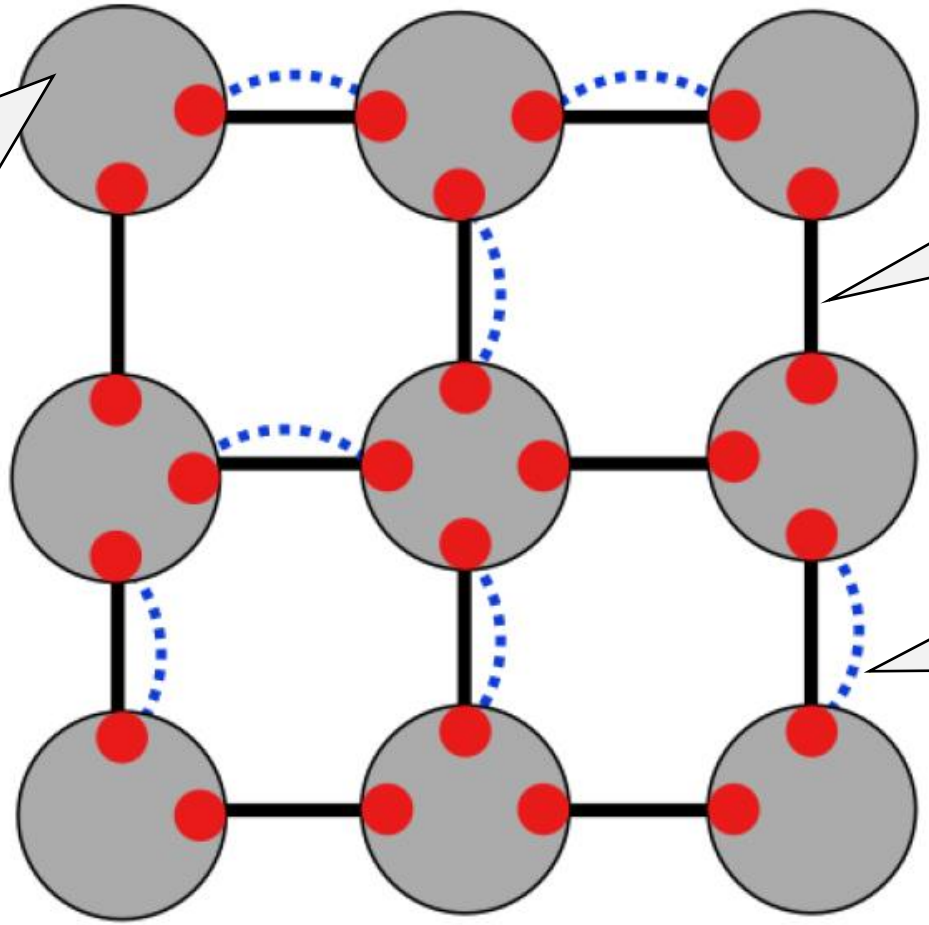

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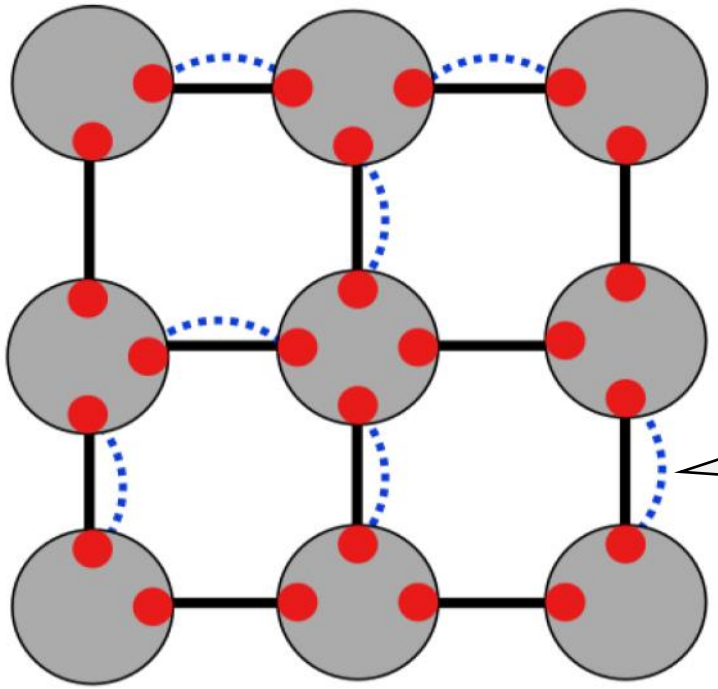
Tc: Quantum memory decoherence time



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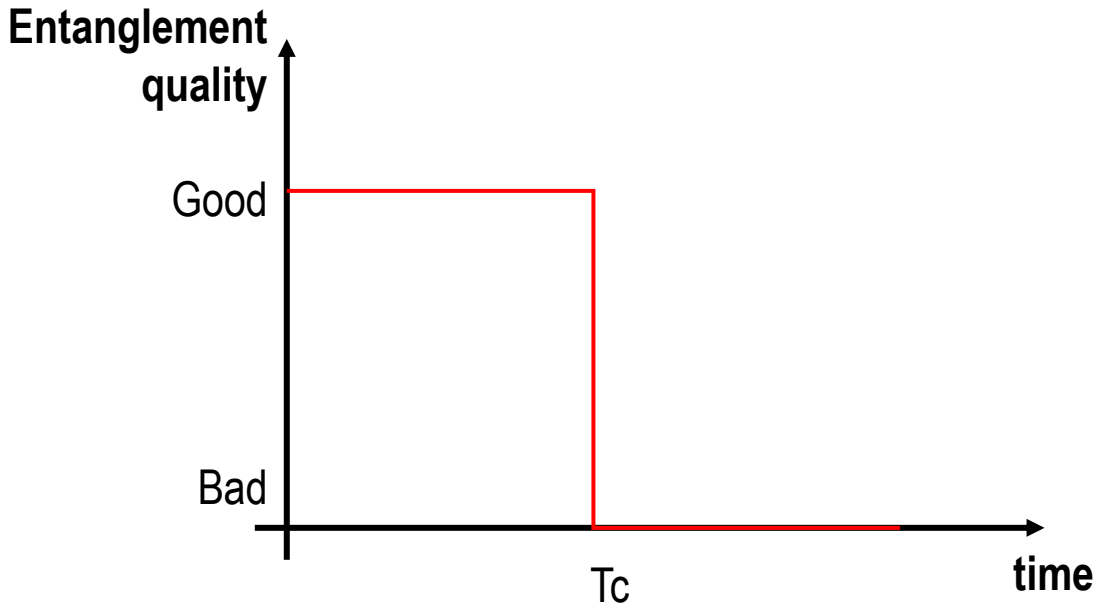
T_c: Time a qubit stored in a quantum memory can maintain entanglement (1)

Cut-off model for Tc



Examples of decoherence times in P. E. Black et al. "Quantum Computing and Communication", Advances in Computers, vol. 56, pp.189-244, 2002

The network model

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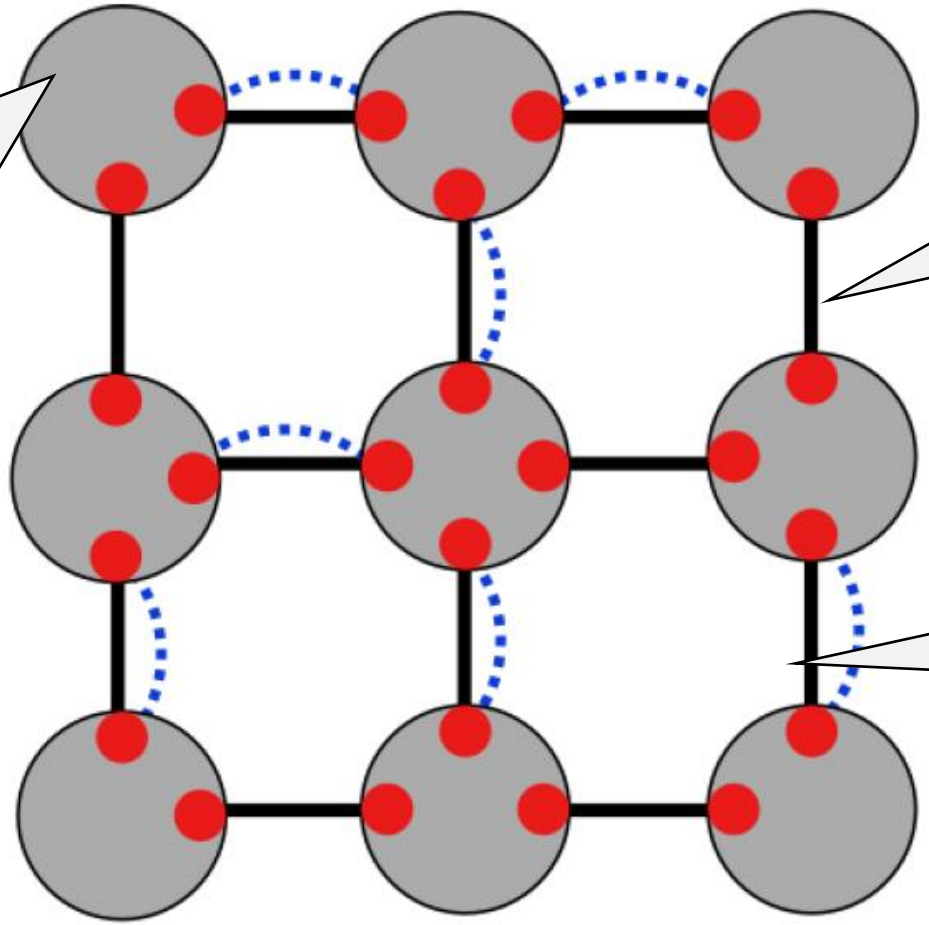

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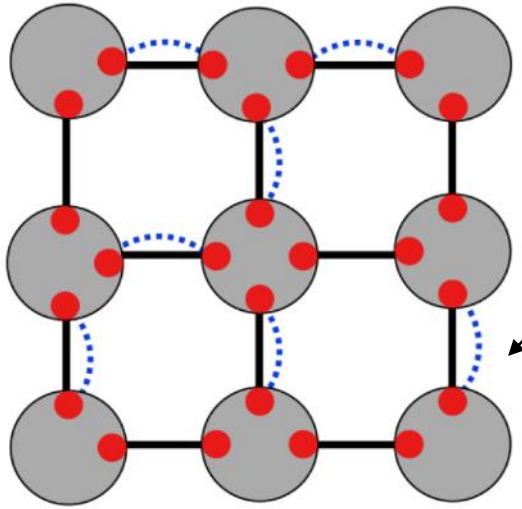
Quantum repeaters

Entanglement swapping: To achieve remote entanglement between two nodes



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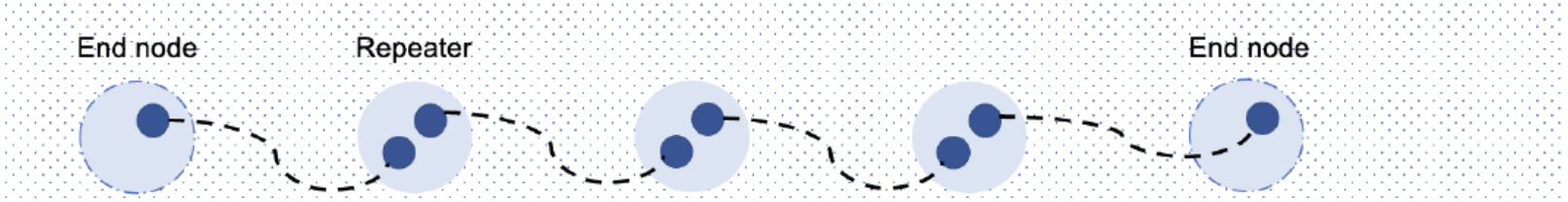

Image: Natasha Siow, MSc student from Quantum Technologies @UCL

Quantum repeaters

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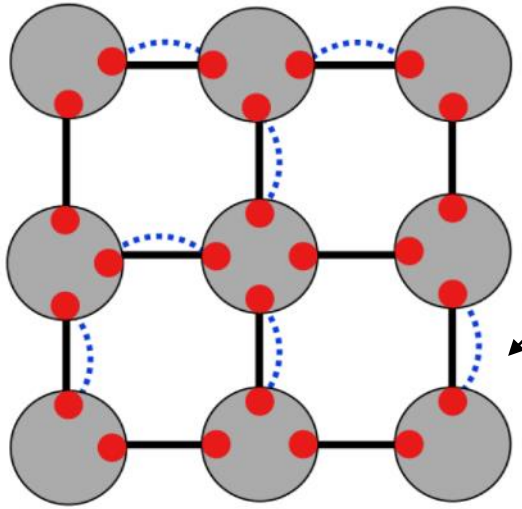

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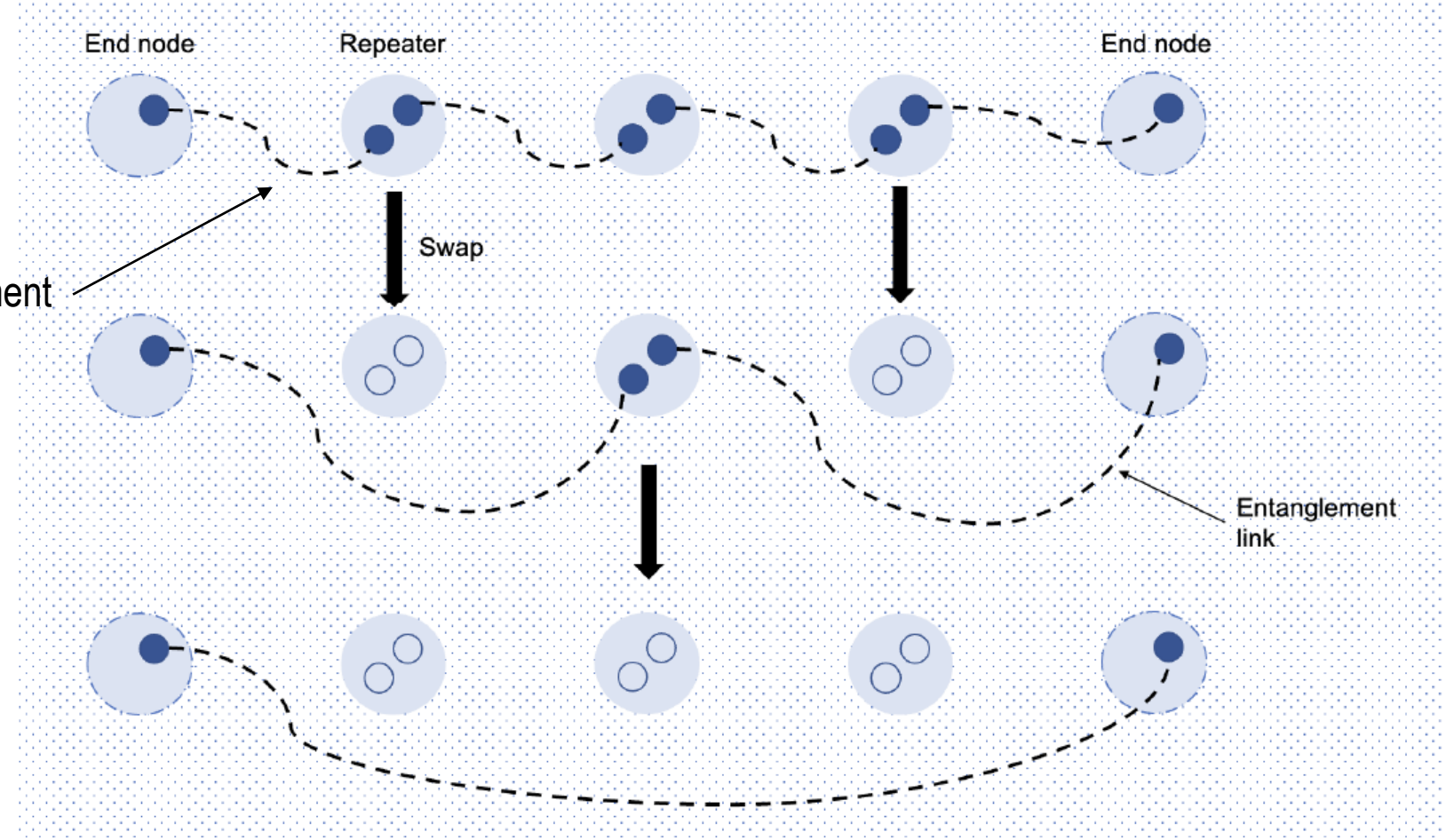

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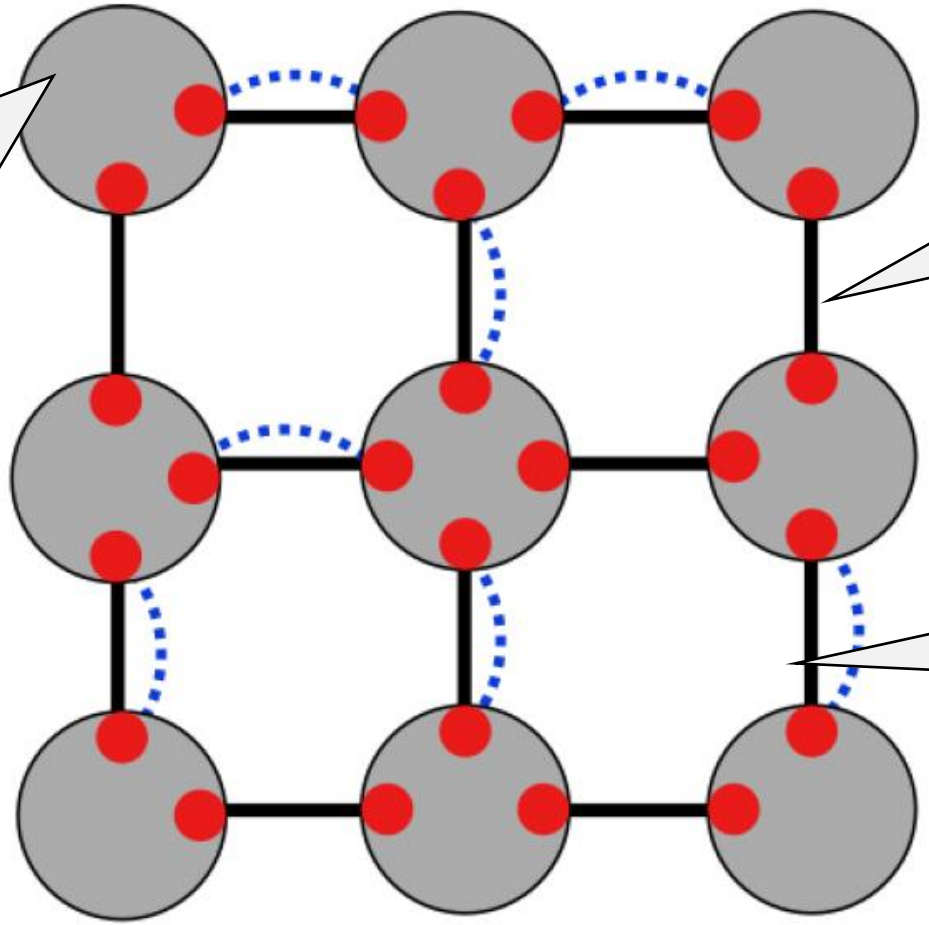

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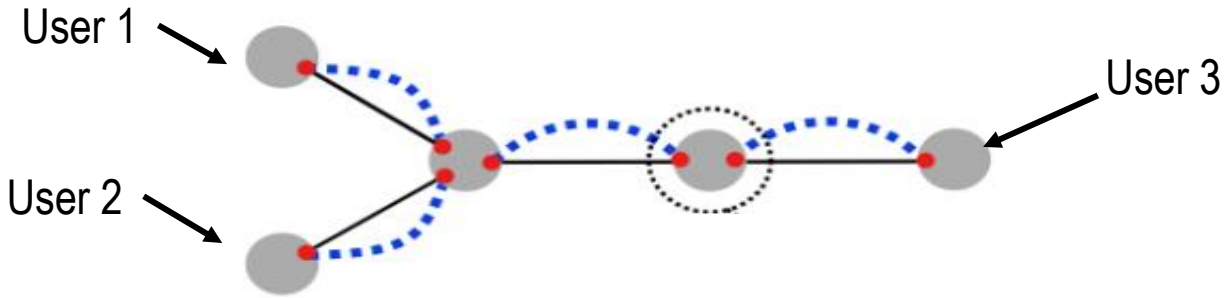
Entanglement fusion

To generate a multipartite entanglement state



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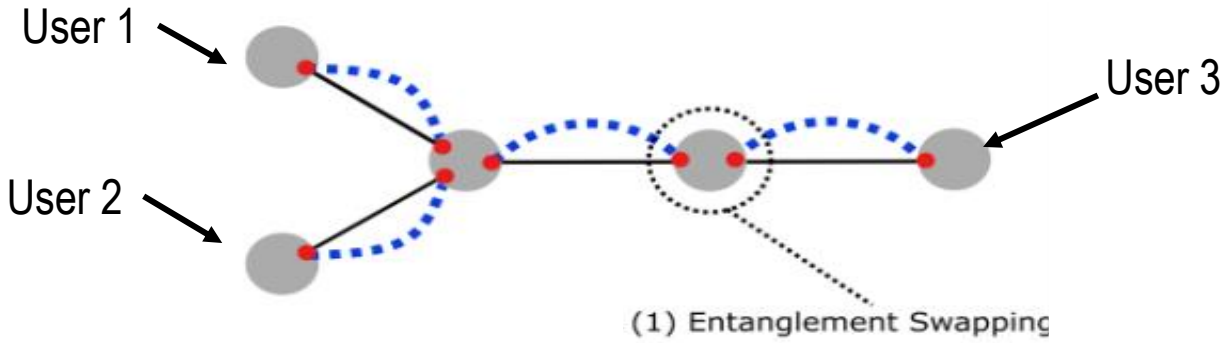

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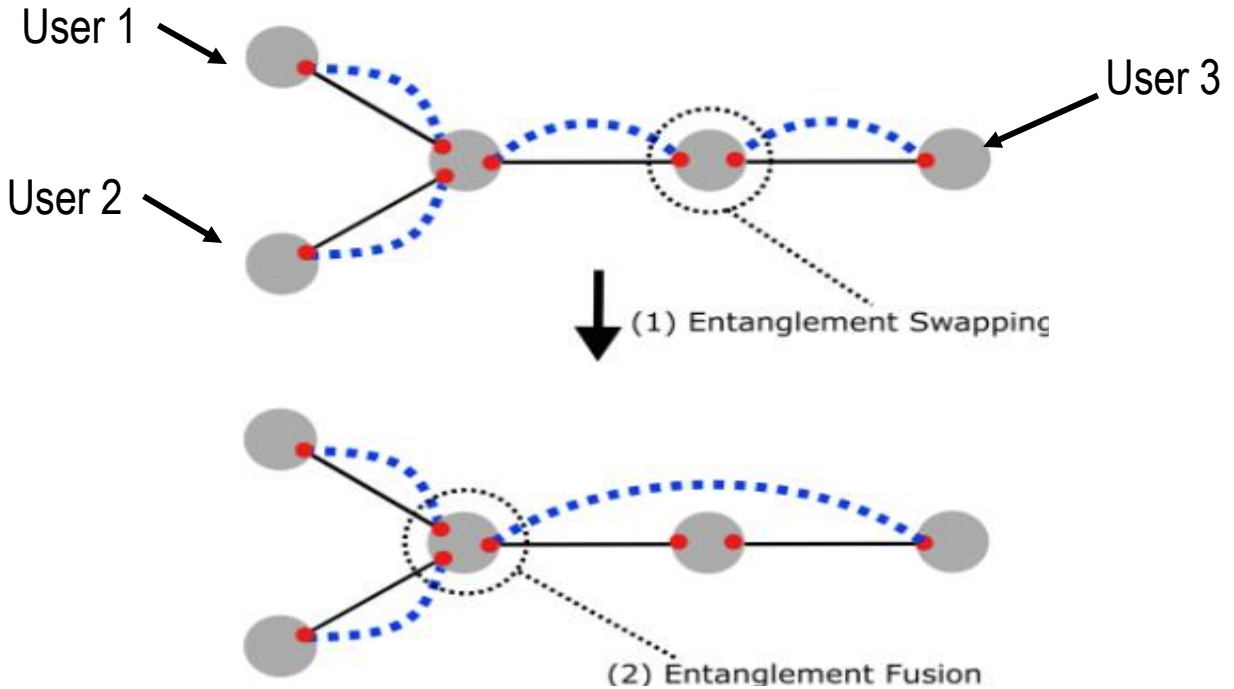

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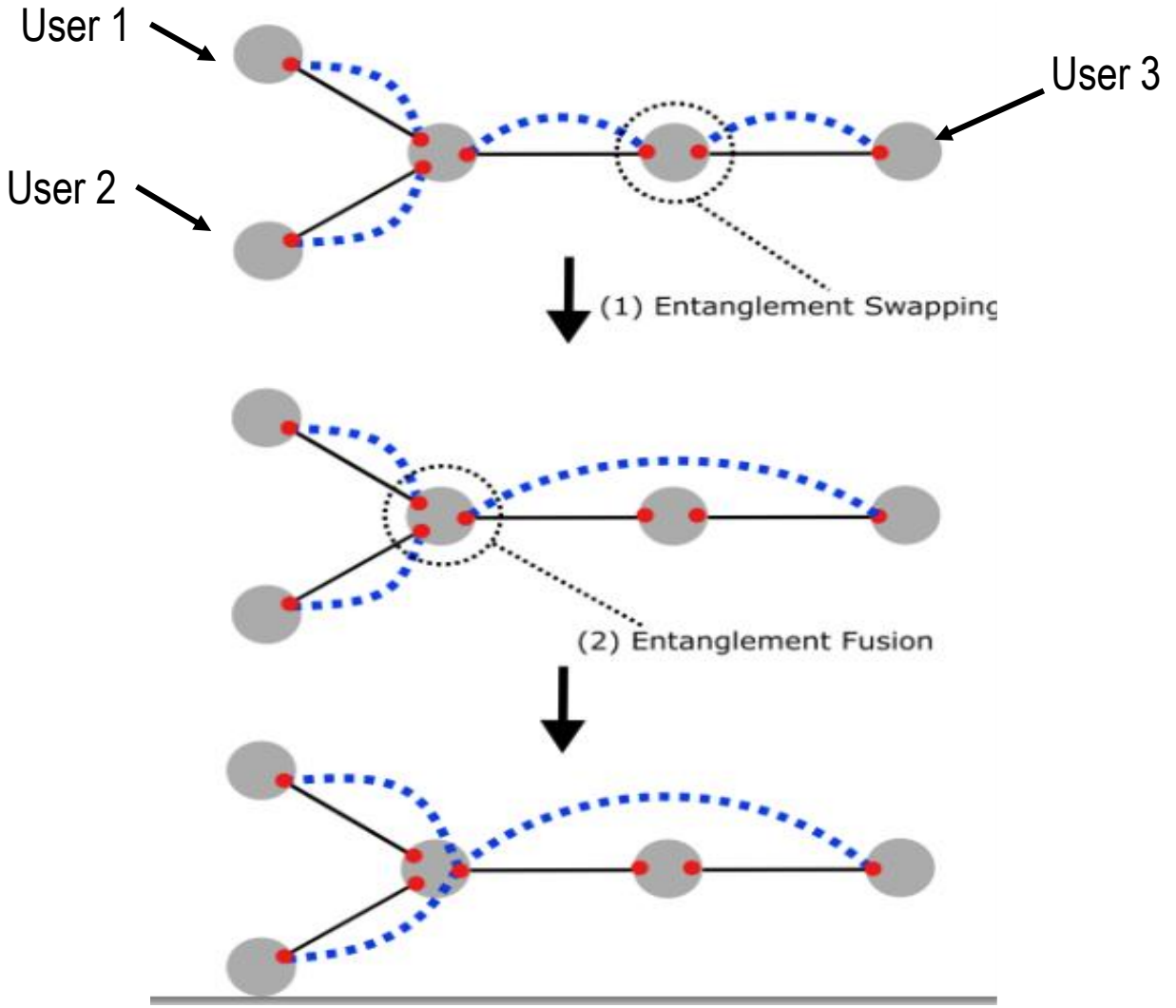


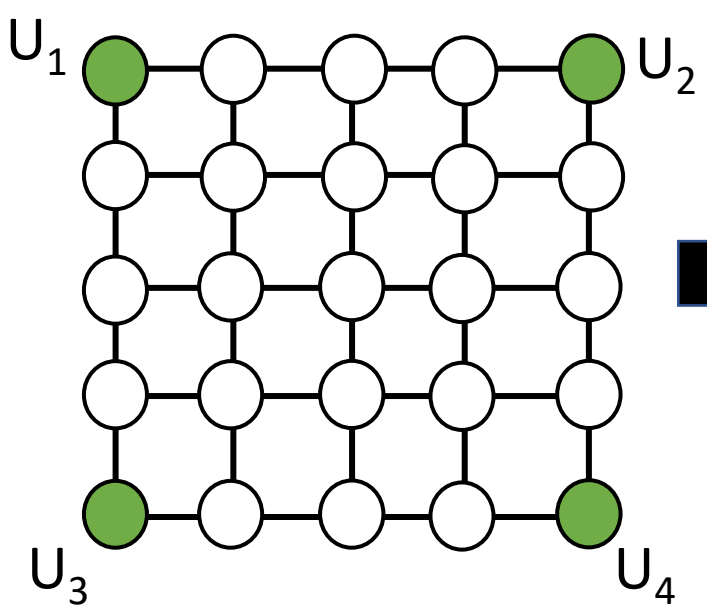
The problem

INPUT:

1. Network topology
2. Set of users U

OUTPUT:

- Routing solution
- Swap & fusion points



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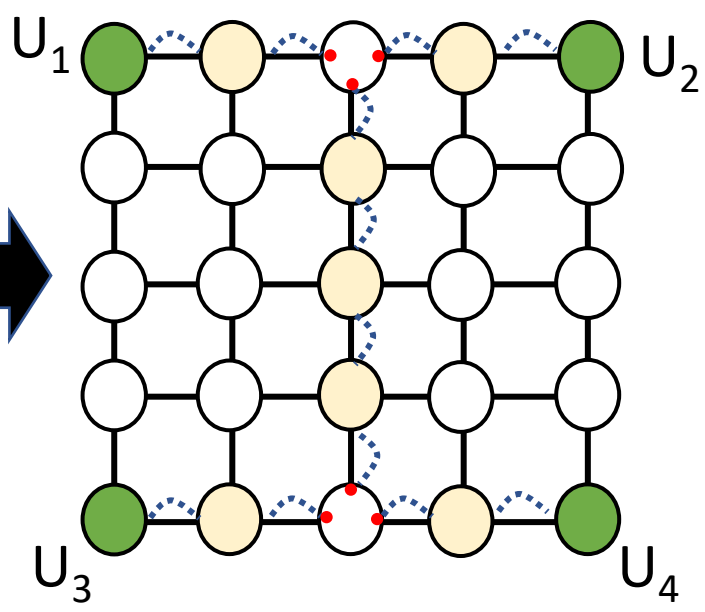
ENTANGLEMENT DISTRIBUTION PROTOCOL



Previous work (SP)

INPUT:

1. Network topology
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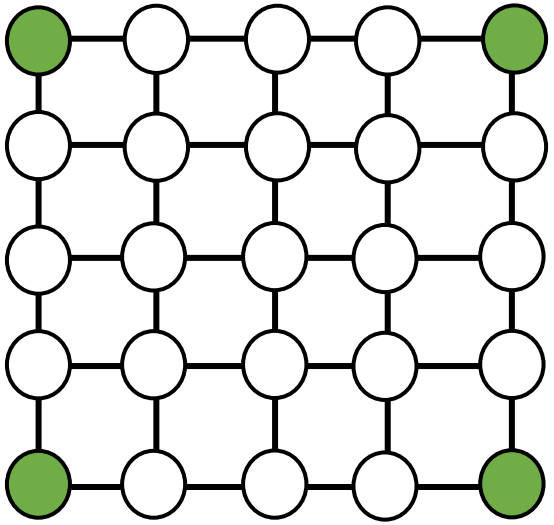

1. Establish a central node
2. Attempt remote entanglement - on **precalculated** edge-disjoint routes
3. Perform fusion at the central node

ENTANGLEMENT DISTRIBUTION PROTOCOL

OUTPUT:

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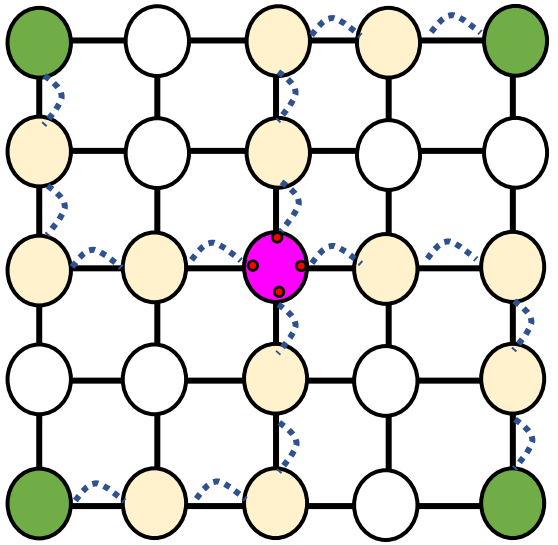

Proposal 1: MP-G

INPUT:

1. Network topology
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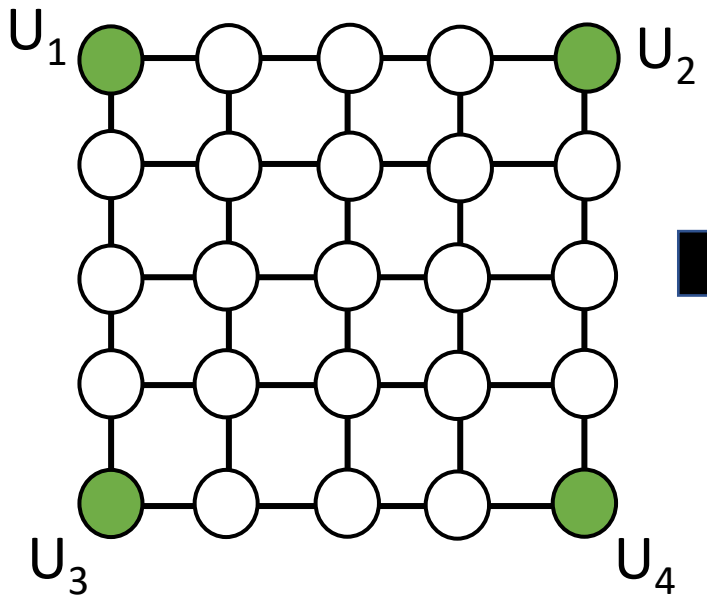

1. Establish a central node
2. Create entanglement links
3. Attempt remote entanglement - on edge-disjoint routes calculated **on the fly**
4. Perform fusion at the central node

ENTANGLEMENT DISTRIBUTION PROTOCOL

OUTPUT:

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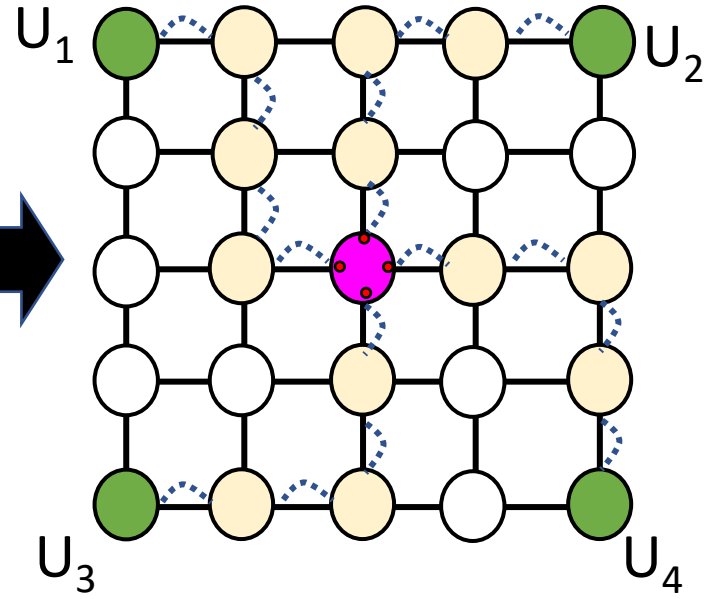
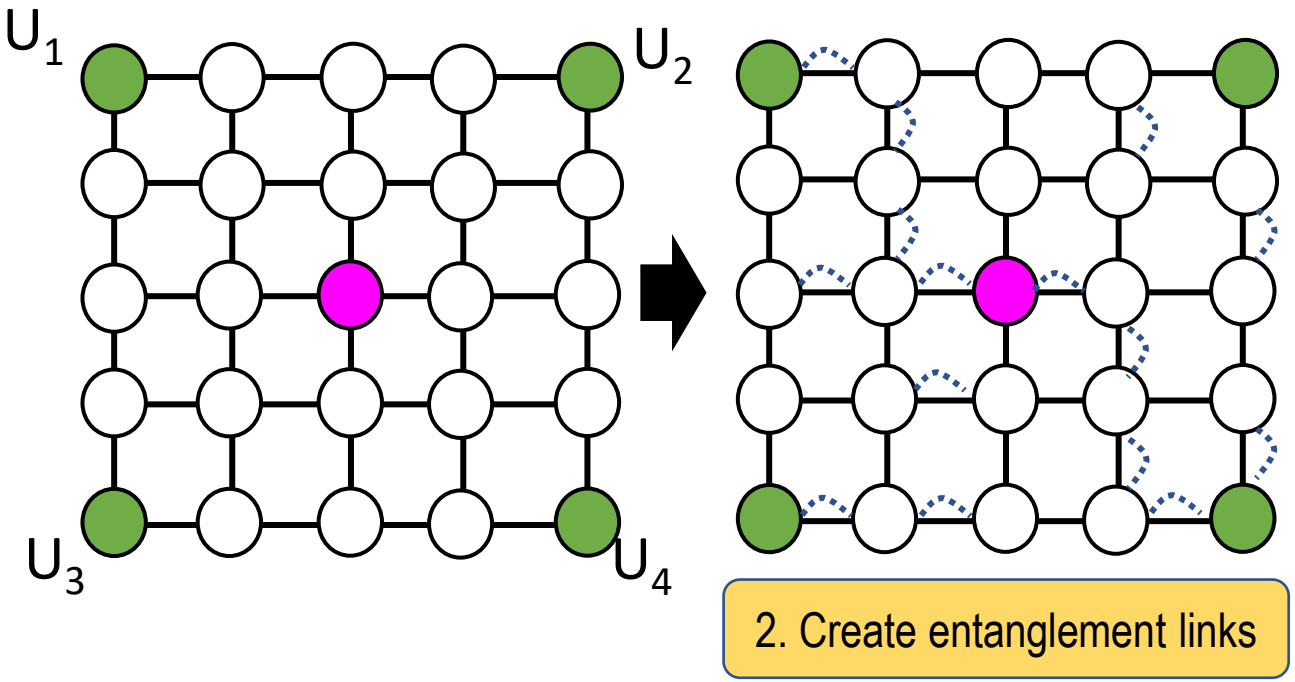


Execution example (T_c=2 time slots)



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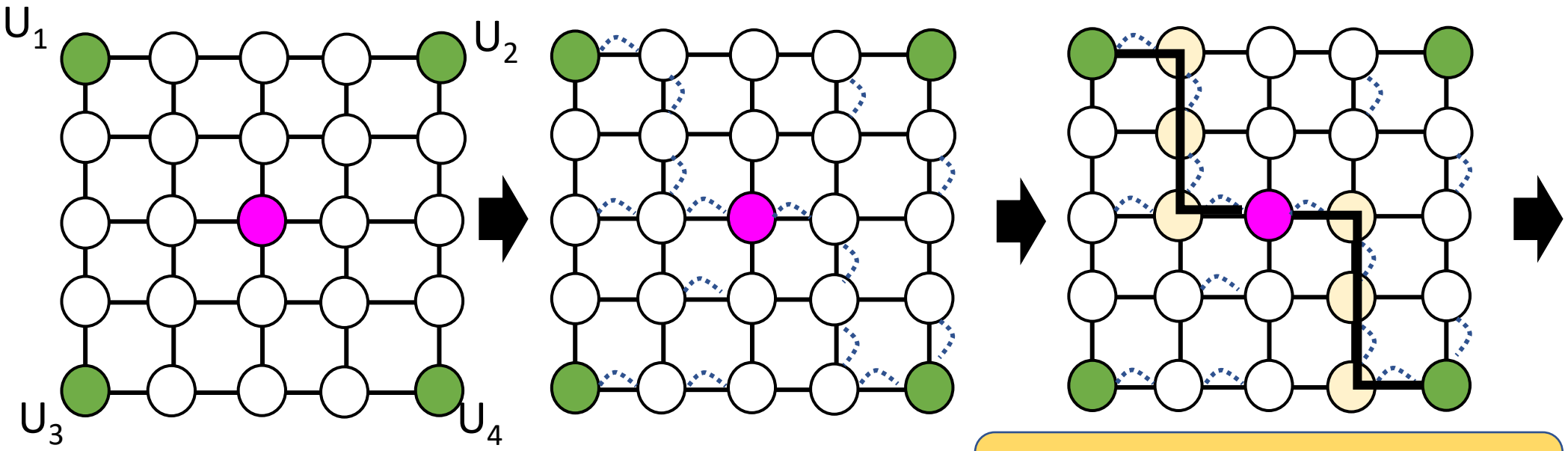

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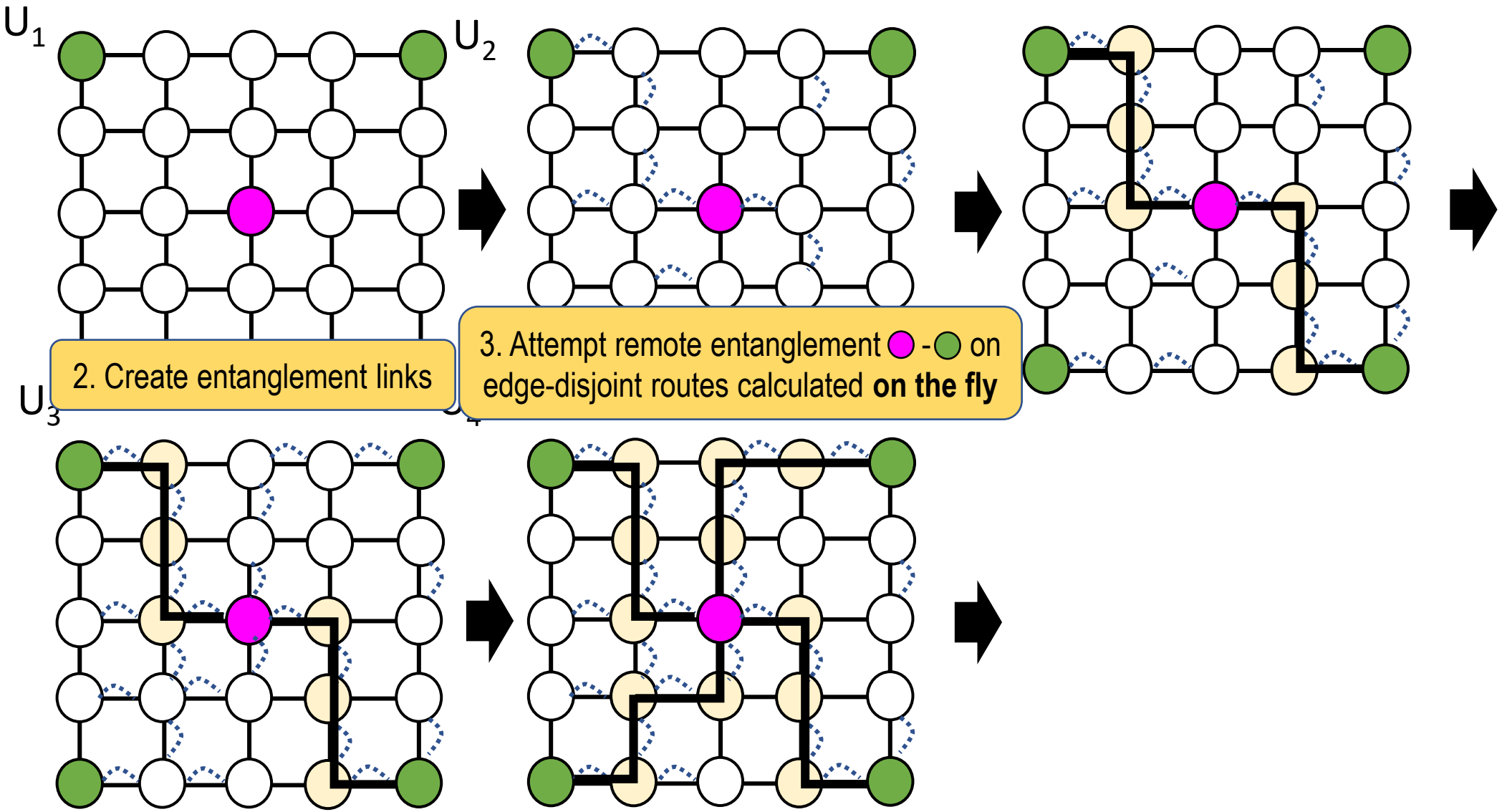

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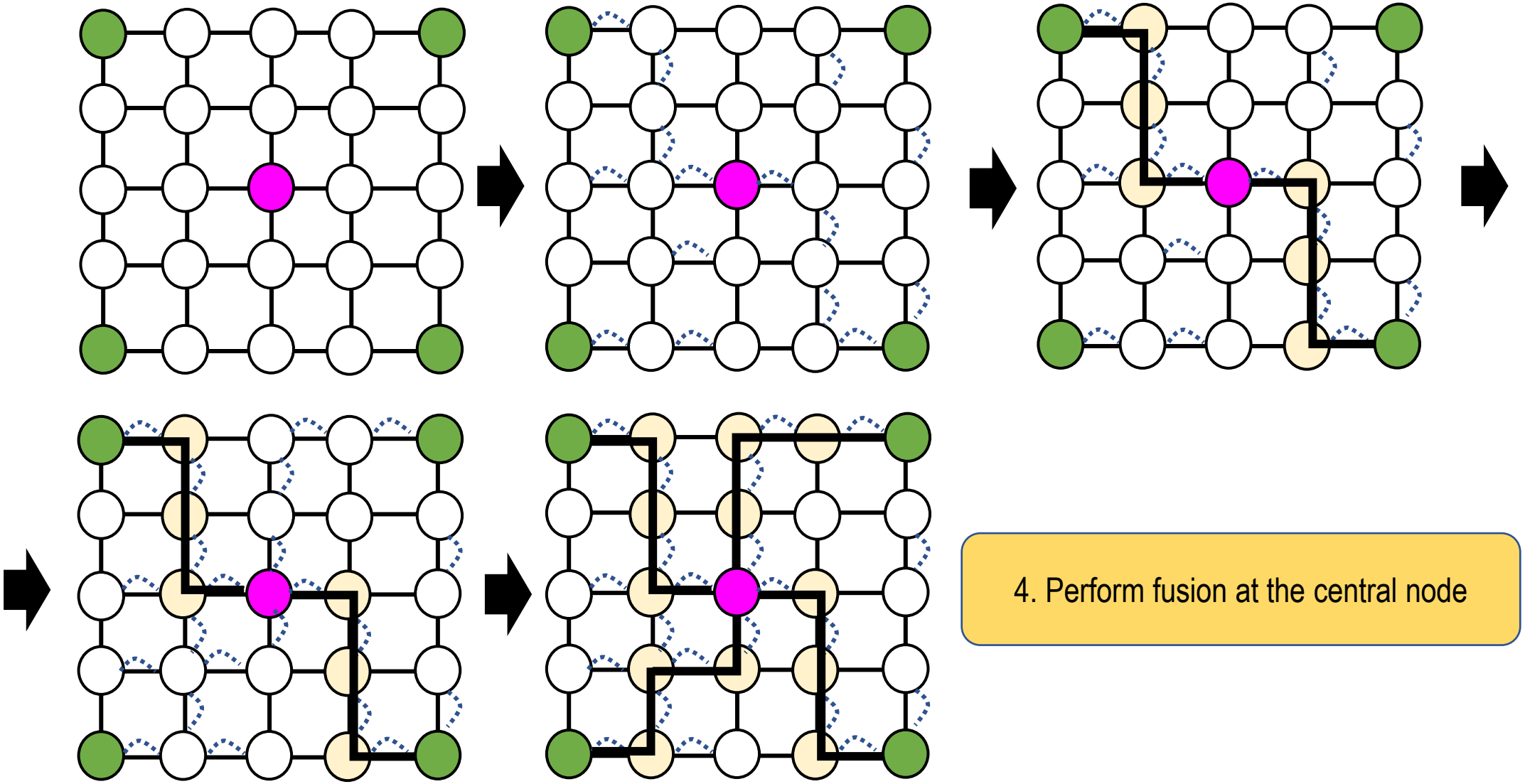

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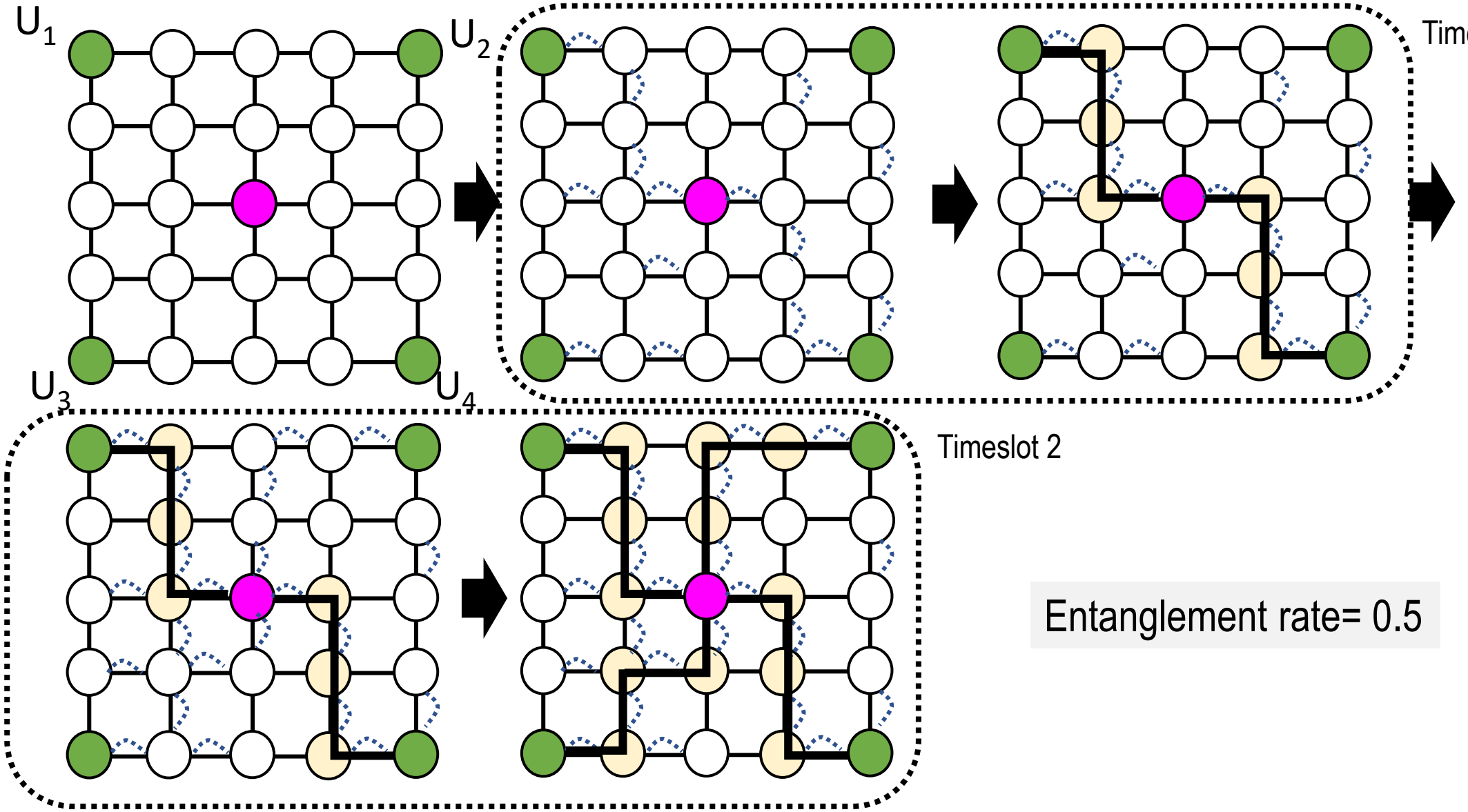

Entanglement rate= 0.5

Proposal 1: MP-C

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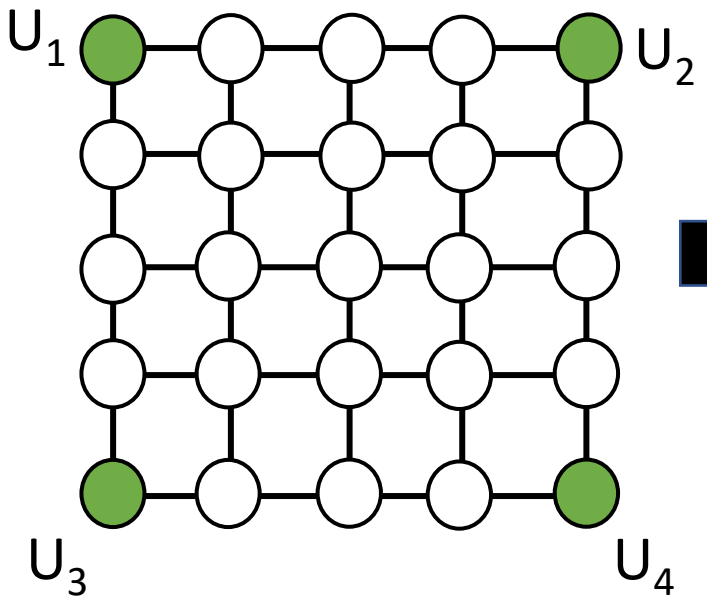

1. Create entanglement links
2. Calculate the Steiner tree covering all users
3. When tree ready, swapping & fusion (fork nodes)

ENTANGLEMENT DISTRIBUTION PROTOCOL

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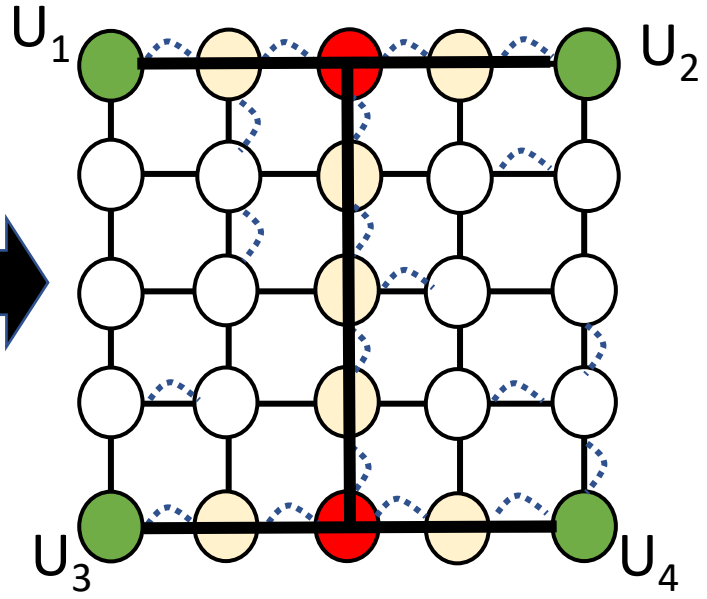

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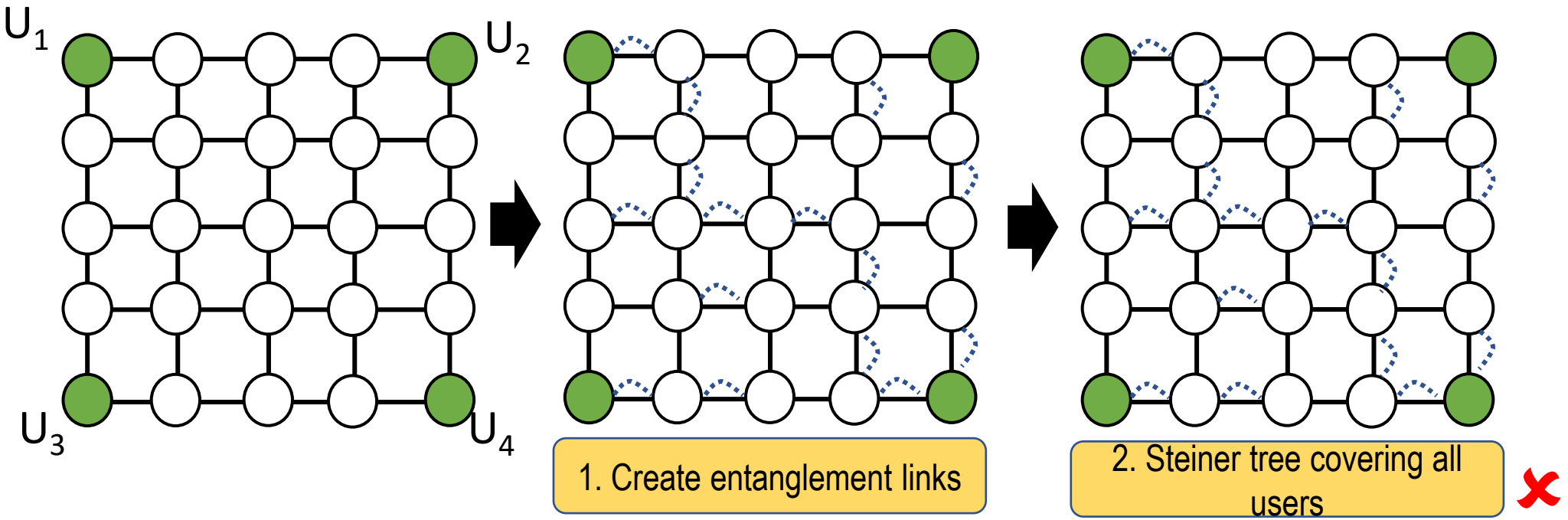

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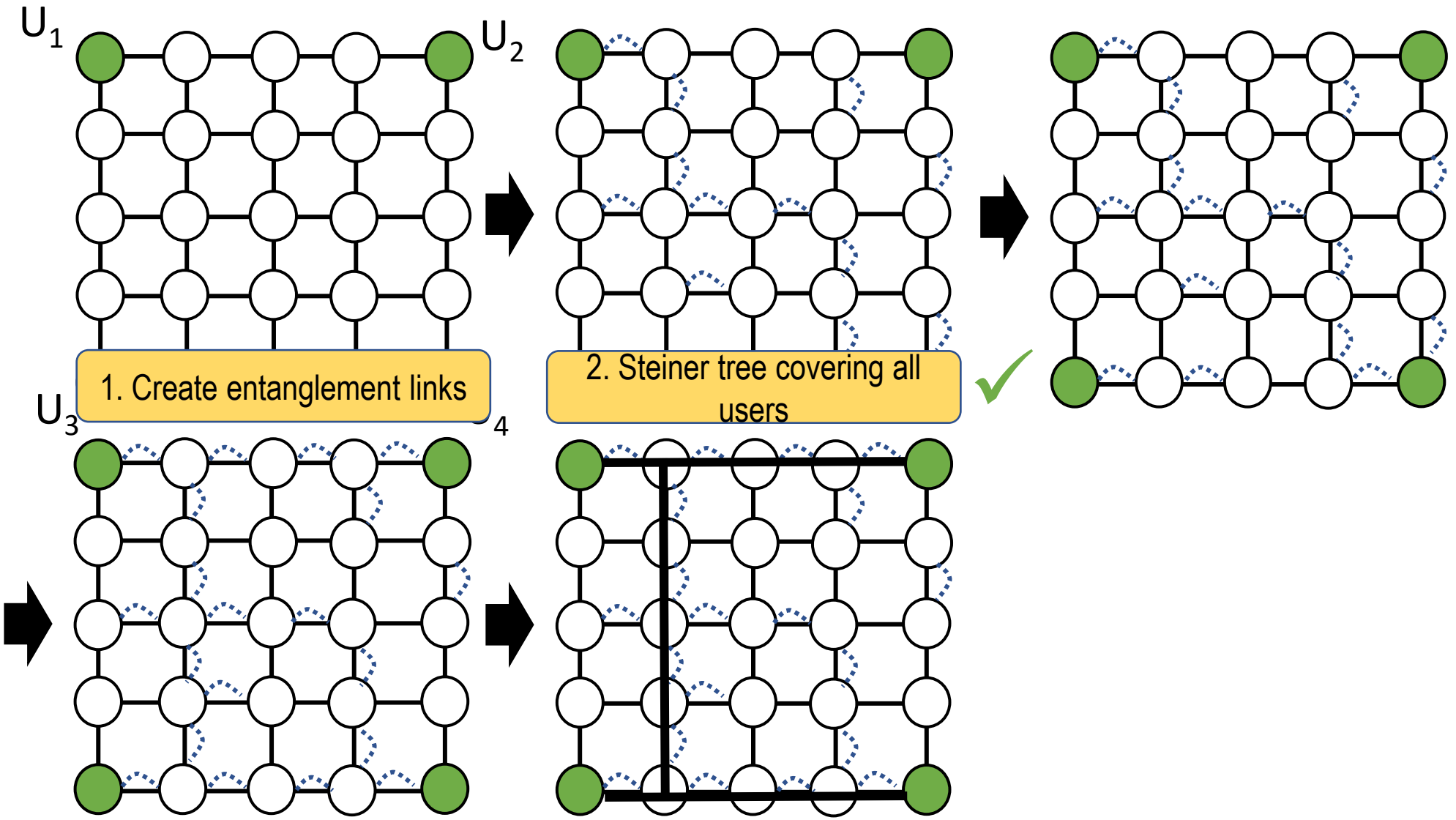

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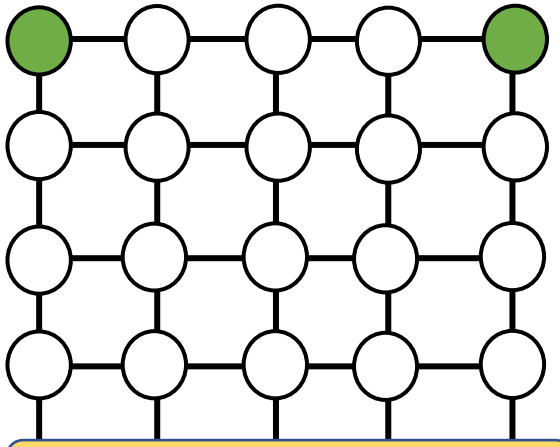

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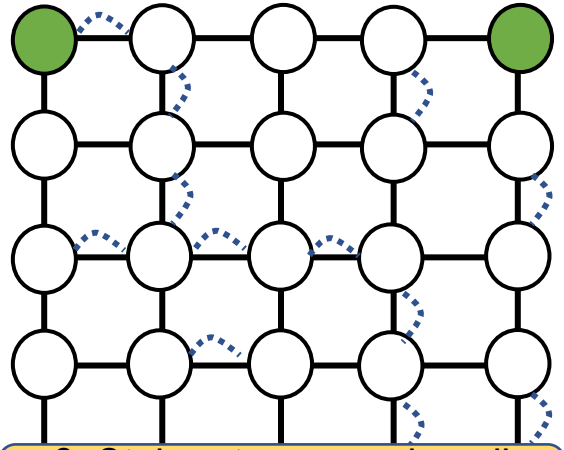

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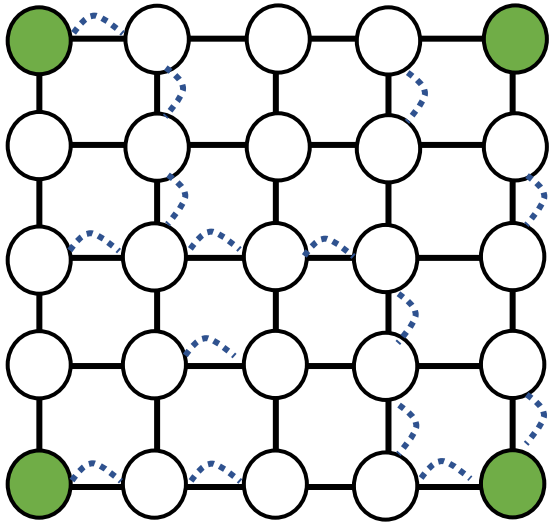

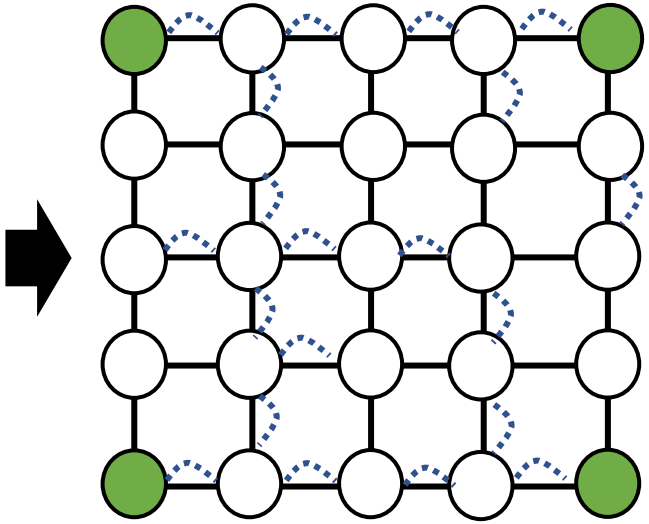

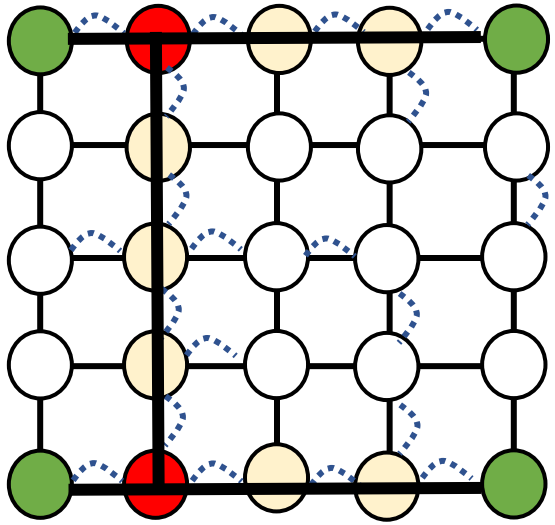

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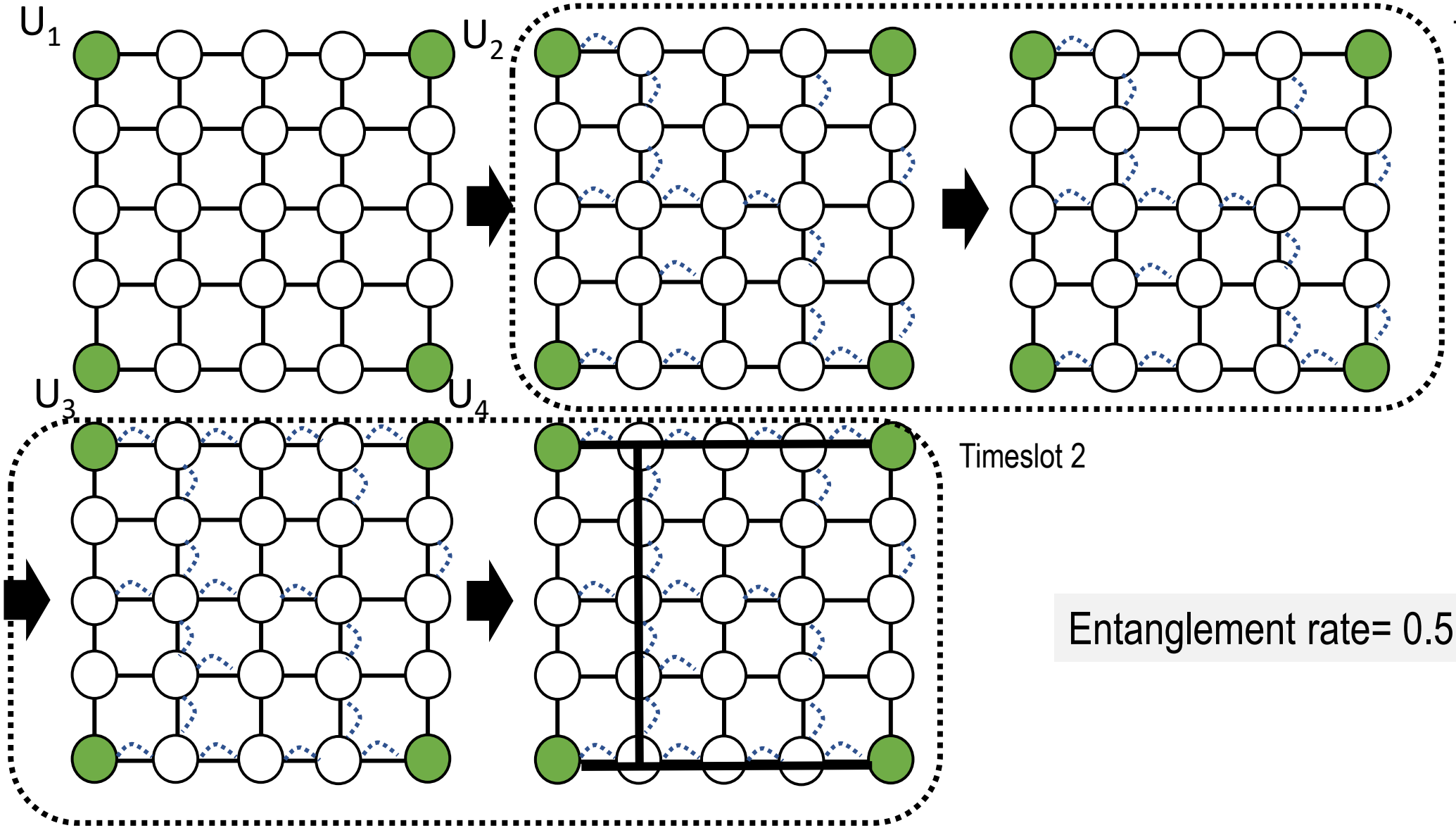
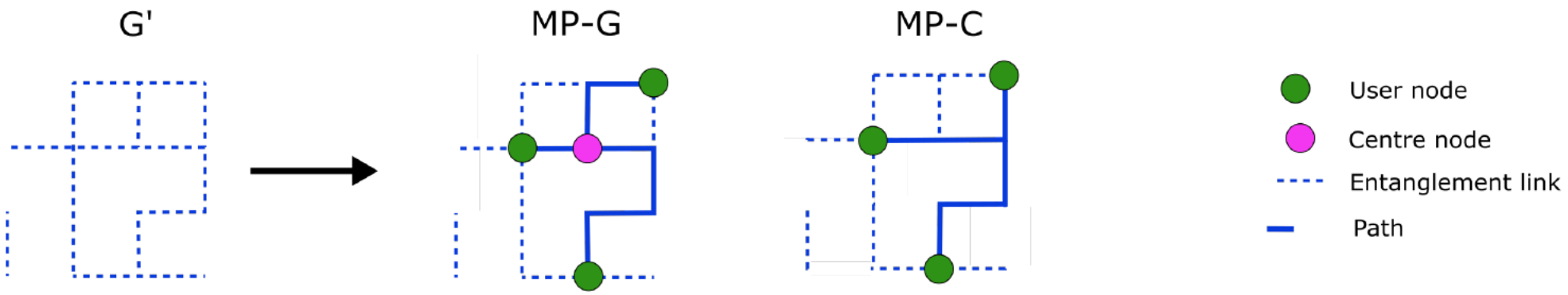


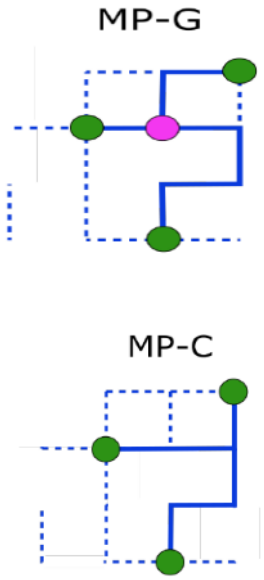
Entanglement rate= 0.5

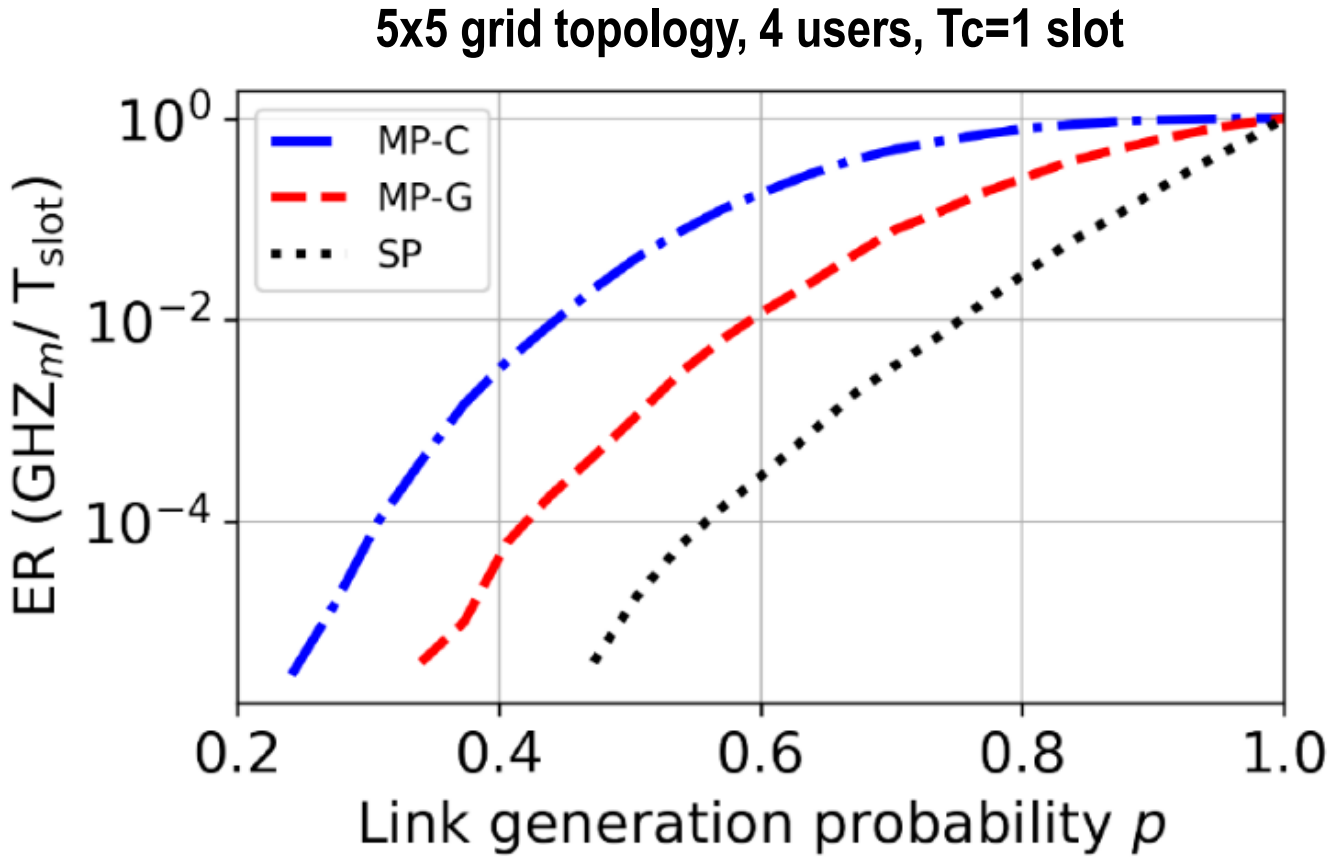
Your guess

Which of the 2 proposed protocols achieve a higher entanglement rate?



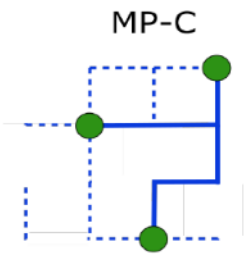
Simulation results: Grids I



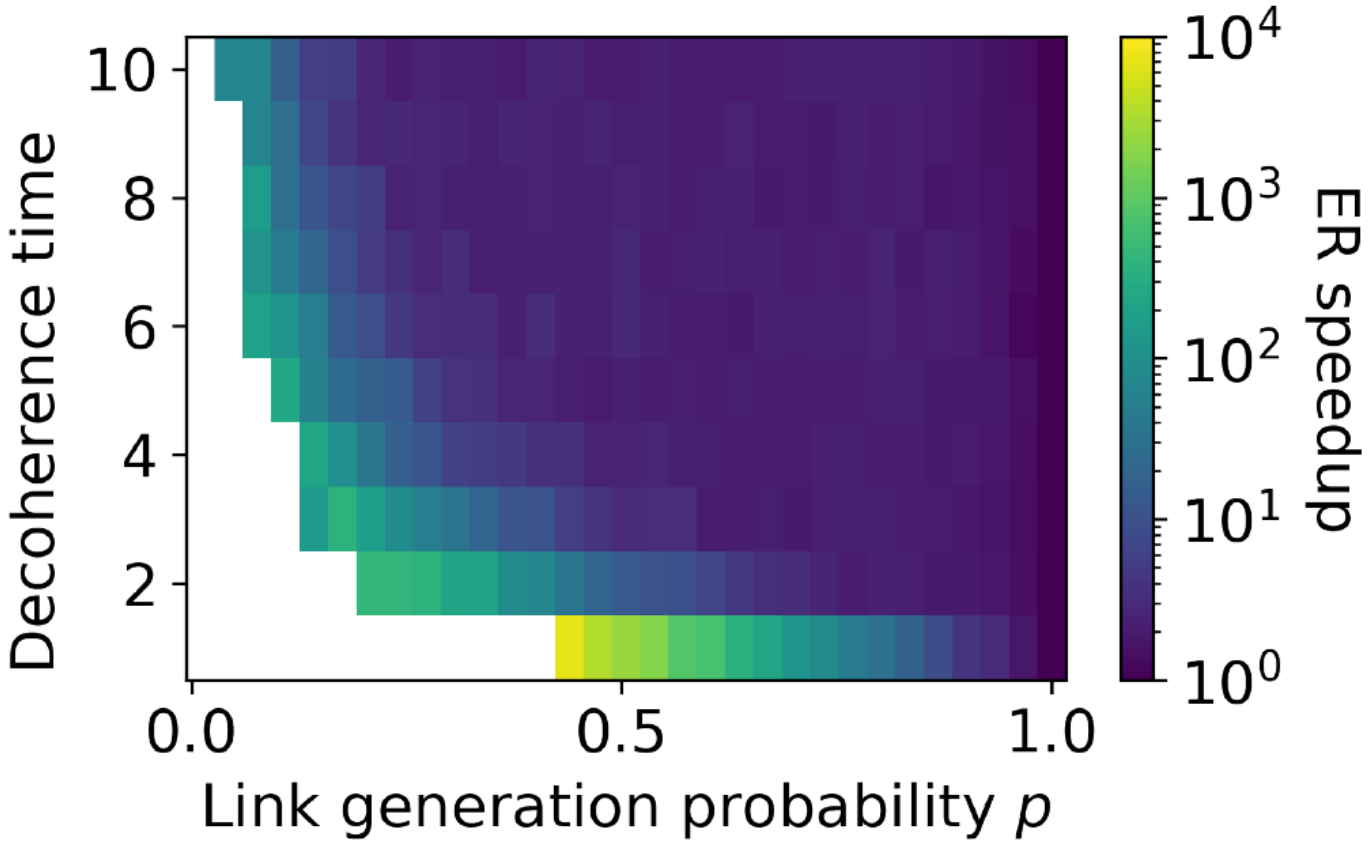


Significant entanglement rate improvement using multipath routing

Simulation results: Grids II



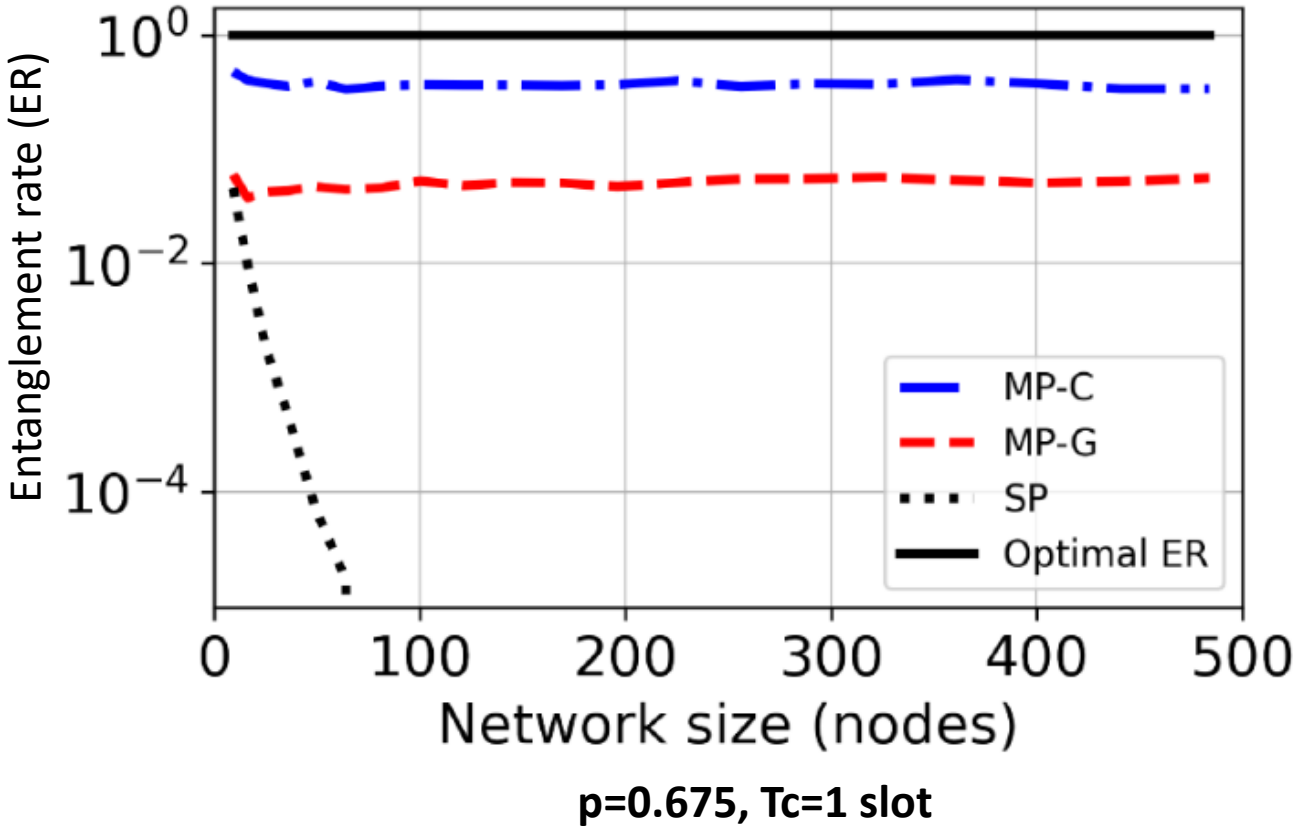
Speed up of MP-C over SP



5x5 grid topology, Tc=1 -10 slots

Higher gains where most needed

Simulation results: Grids III

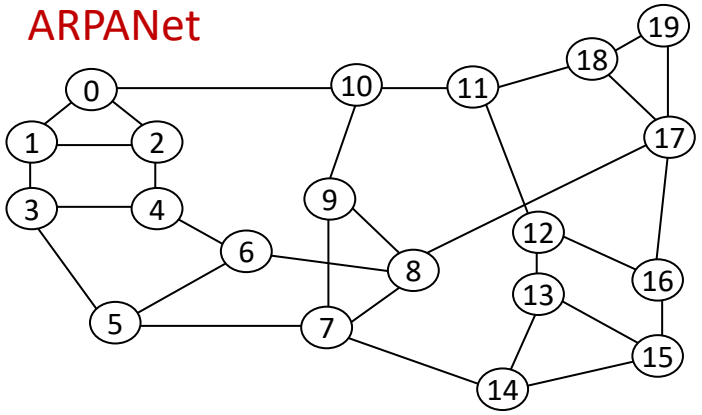


p=0.675, Tc=1 slot

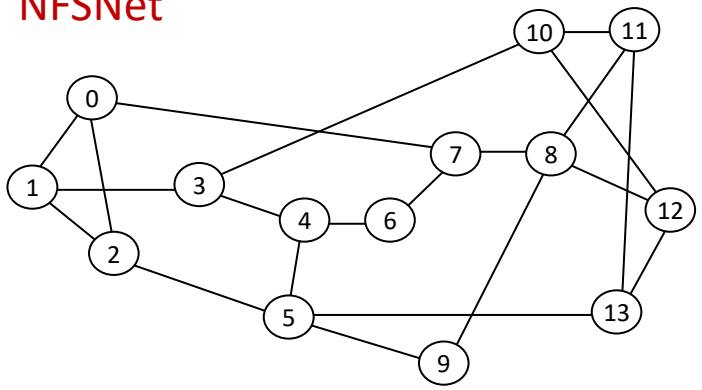
- Multipath routing entanglement rate **independent of network size**
- ER for shortest path routing decays exponentially

Simulation results: Mesh topologies

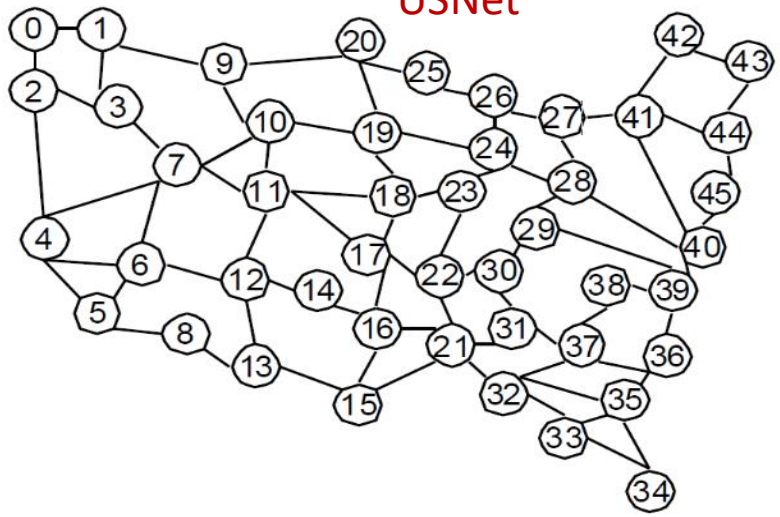
ARPANet



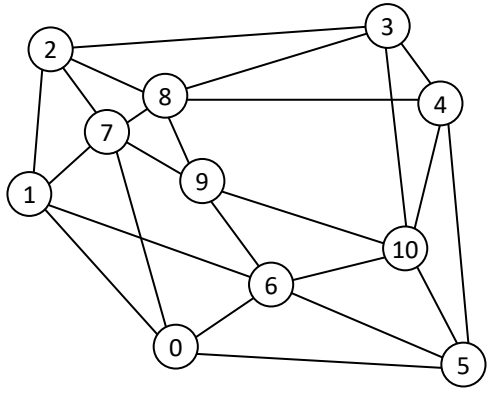
NFSNet



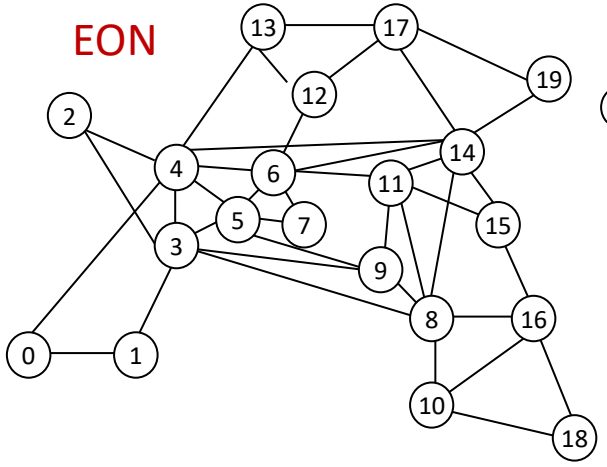
USNet



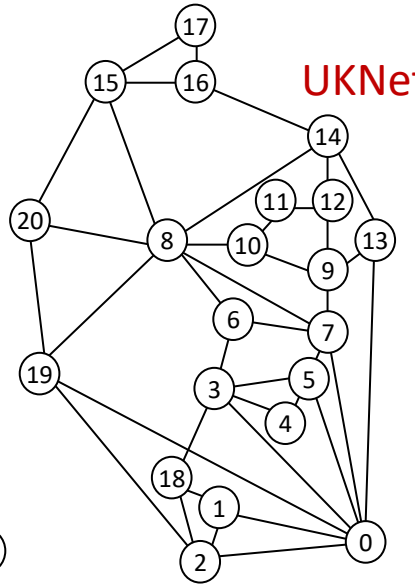
Eurocore



EON

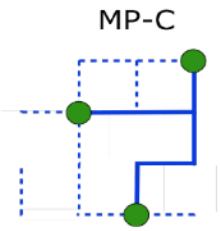


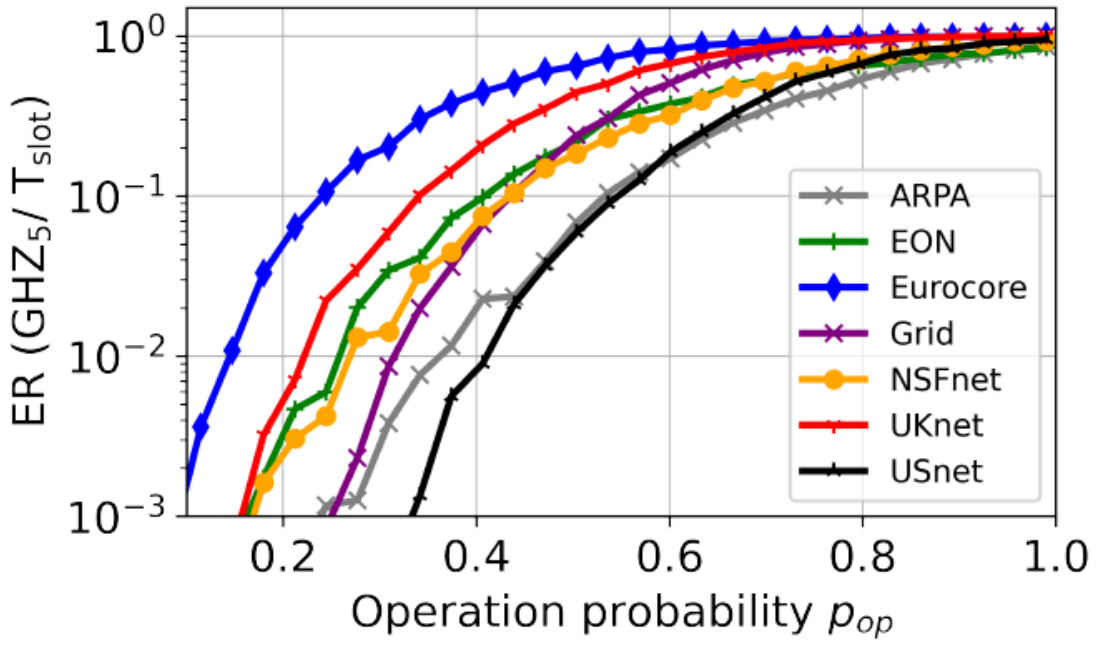
UKNet



Network Name	Nodes	Edges	Average edge length (km)	Average nodal degree
ARPA	20	31	6.09	3.1
EON	20	39	7.24	3.9
Eurocore	11	25	4.26	4.55
NSFnet	14	21	5.09	3.0
UKnet	21	39	1.38	3.71
USnet	46	76	4.34	3.3
Grid (Fig. 2)	36	60	1.0	3.33

Simulation results: Mesh I





10^4 different 5-user combinations, each attempted for up to 5000 timeslots, Tc=1 slot

- High average node degrees (Eurocore, EON and UKNet) networks achieved higher ERs.

Network Name	Nodes	Edges	Average edge length (km)	Average nodal degree
ARPA	20	31	6.09	3.1
EON	20	39	7.24	3.9 ③
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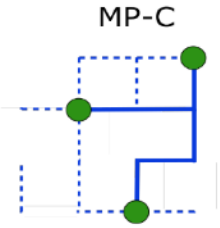
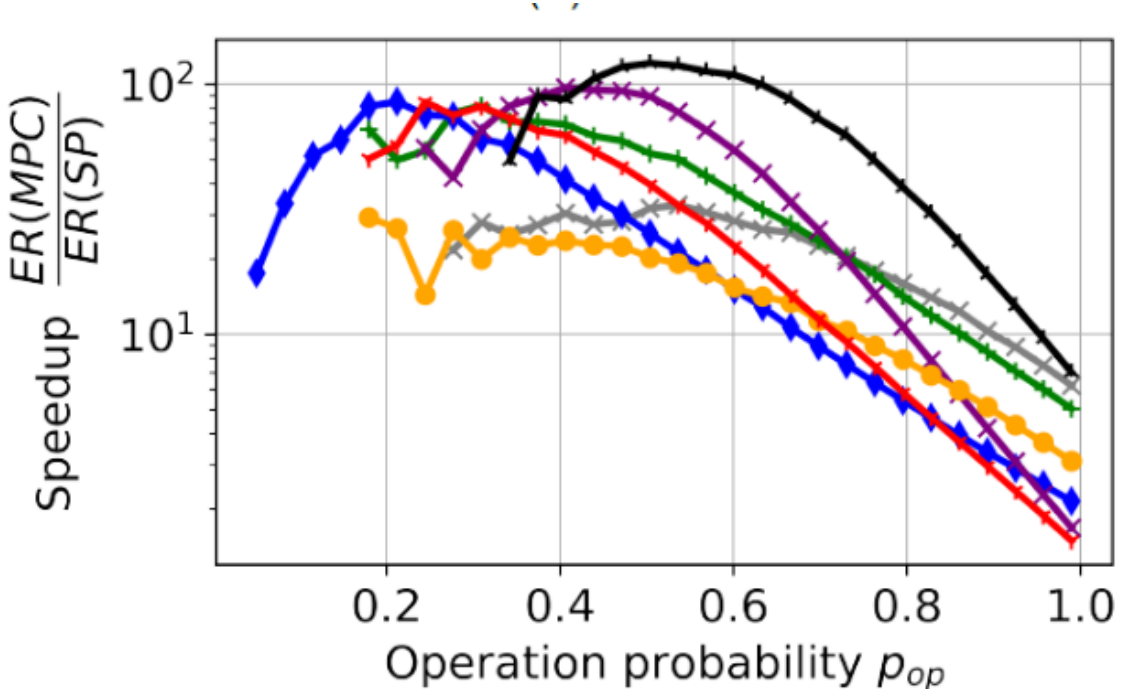
Grid (Fig. 2)	36	60	1.0	3.33

Simulation results: Mesh II

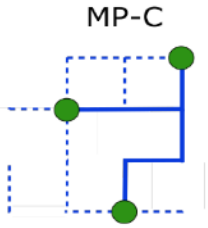


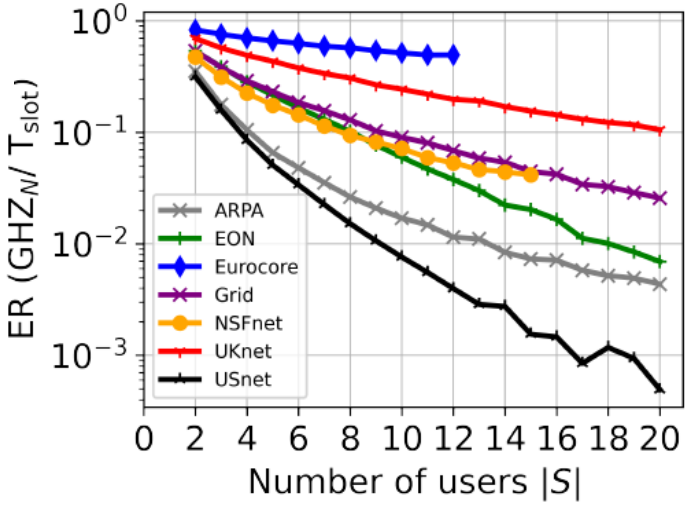


10^4 different 5-user combinations, each attempted for up to 5000 timeslots, Tc=1 slot

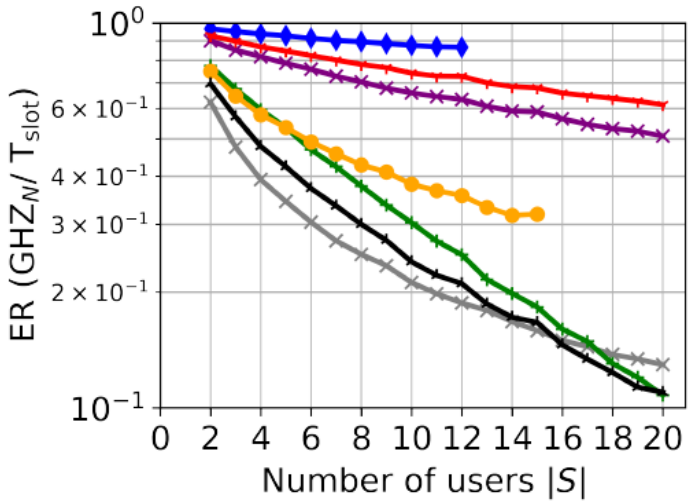
Maximum speedup at intermediate values of p_{op} (varied for different topologies)

Simulation results: Mesh III

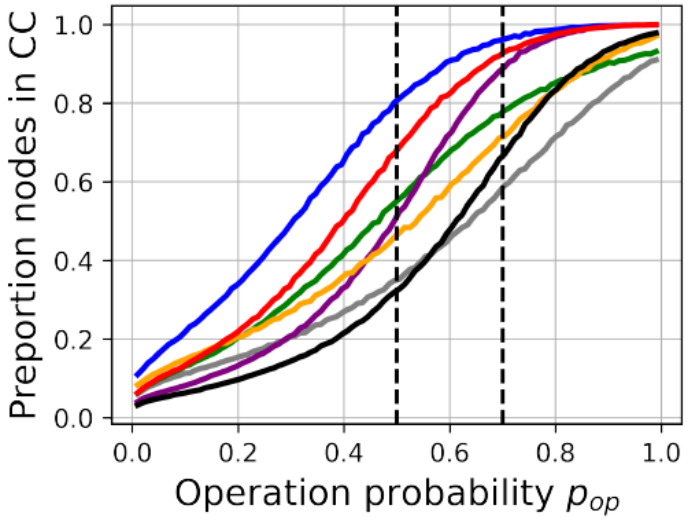




(a) p_{op}=0.5



(b) p_{op}=0.7



(c)

- There is a **critical value of p** at which a high percentage of nodes are in a single connected component.
- This value is **topology-dependant**

Summary

- Multipath **increases ER** several orders of magnitude (NISQ p & T_c)
- Multipath makes **ER size-independent** (for threshold p)
- Multipath **ER is topology-dependant** (nodal degree)